INTRODUCTION TO

RATEMAKING AND LOSS RESERVING *FOR PROPERTY AND CASUALTY* INSURANCE

4th EDITION

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Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance

Fourth Edition

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PREFACE TO THE FOURTH EDITION

This text is designed for use in teaching an introductory course in property/casualty insurance topics. The material can be covered comfortably in a twelve week teaching semester with time for review and problem classes. The material also represents the basic foundation of knowledge needed to gain an introductory appreciation of the building blocks of the property/casualty actuarial discipline.

Although the material presented here is purely property/casualty in its origins, the application of the knowledge gained is much broader. The methods presented here have potential application not only in the property/casualty practice area, but also in accident and sickness insurance, group life and health insurance, and many other related areas.

Students of this introductory material should not, however, presume to have gained a working knowledge in the property/casualty practice area. This book is designed to provide an overview of the core principles and methodologies, incorporating examples as an educational tool. To achieve a proper working knowledge in this field, the interested reader is advised and encouraged to pursue the broader base of material available through a more advanced actuarial syllabus.

Some time ago, I was approached by Gail and Rob and asked to be involved in updating this text. Having reviewed the third edition of this text and taught a fourth year property and casualty university course since 2009, I was only too happy to get involved in this project. I know many actuaries who have found this text useful as an introduction for students new to the property and casualty actuarial profession. With that in mind, my intention was to keep this material at the introductory level, while leaving the more advanced concepts and methods to a more robust actuarial curriculum.

The fourth edition reverses the order of chapters three and four from previous editions. The estimation of the ultimate claim payments is a necessary first step in both the loss reserving process and ratemaking process. Determining the ultimate losses is more comprehensively covered

in the loss reserving chapter, and the ratemaking process often relies on the estimates of ultimate losses determined in the loss reserving process. As a result, the loss reserving chapter now comes before the ratemaking chapter.

The frequency and severity section of the loss reserving chapter has been revised to more fully demonstrate the closure method of estimating ultimate losses.

Chapter Five has been completely rewritten and updated to include deductible pricing, as this alternative approach to the ratemaking in Chapter Four is typically used for pricing various deductible options.

Finally, the fourth edition has also been updated to reflect industry changes over the past seven years and includes even more additional exercises.

February, 2015
W. Scott Lennox, FSA, FCIA, FCAS

WHY INSURANCE? ○ 1

1.1 THE EVOLUTION OF INSURANCE

Humans have strived for security since the beginning of their existence. At its earliest point, security existed if there was an assurance of food, warmth, and shelter. The Bible relates the story of how, in ancient Egypt, Joseph set aside part of the crop in good years in an attempt to cover the expected shortfall in years of drought.

> The World Bank has recently identified casualty (or general) insurance as a critical element for the development of emerging economies.
>
> This is only the latest recognition of the importance of casualty insurance to economic development. The roots of insurance can be traced back to Babylonia, over four thousand years ago, when traders developed markets to insure the goods on their caravans against loss on the hazardous trade routes. Without this form of property insurance, traders would have been reluctant, or financially unable, to engage in the trade that led to this nascent western civilization. Recognized as the oldest branch of insurance, marine insurance was developed in ancient Greece and enabled trade to occur and civilization to flourish. Again, forms of casualty insurance were the essential ingredients to economic development. The lack of life insurance on the captain, or a pension system for the sailors, did not stop ships from sailing. But without insurance on the ships and cargo, trade stopped.[1]

As society developed and the roles of individuals within the economic framework became more specialized, the need for economic security increased.

Economic security is the opposite of ***economic risk***, which we will refer to simply as ***risk***. Risk derives from variation from the expected, not from probability. For example, on a cloudy morning we may be told there is a

[1] From Steve D'Arcy, CAS President, *The Actuarial Review*, Nov 2005, p. 7.

risk of rain. What is meant, more correctly, is that there is a high probability of rain. The variation associated with the weather forecast could be just as high or higher on a sunny morning.

A modern industrial society provides many examples of risk. A homeowner faces a large variation associated with the potential economic loss caused by a house fire. A driver faces a similar, though less variable, potential economic loss if his or her car is damaged. A larger possible economic loss would be associated with the injury of a third party in a car accident for which you are responsible.

Examples of early informal insurance arrangements can be found in the cooperatives and fraternals that existed in Europe over 400 years ago. For example, the farmers in a certain area would agree, usually informally, that if one farmer's barn was destroyed, the community would see that it was rebuilt. If the breadwinner in a family unit died, the community would "pass the hat" to establish a fund for the surviving dependents. In this informal arrangement, each person's economic risk was shared or pooled among the members of the community.

These informal systems proved to be adequate for several hundred years. At the time of the industrial revolution, however, the need for a more formal system arose. Because of the rapid urbanization of the population, it became true that one's neighbor could be a stranger with whom one had no interests in common. Hence, it was no longer sufficient to expect a communal or cooperative response when one family unit met with an economic reversal.

It was perfectly natural that the "pooling" concept of the existing cooperatives and fraternals became formalized in the new insurance industry. Under the new formal arrangement, each policyholder still implicitly pooled his or her risk with all other policyholders. However, it was no longer necessary for any individual policyholder to know or have any connection with any other policyholder.

1.2 HOW INSURANCE WORKS

If we look at the risk profile of an individual, we see that there is an extremely large variation of possible outcomes, each with a specific economic consequence. Thus, any individual is exposed to a significant amount of risk associated with perils like death, fire, disability, and so on.

By purchasing an insurance policy, an individual (the ***insured***) can transfer this risk, or variability of possible outcomes, to an insurance company (the ***insurer***) in exchange for a set payment (the ***premium***). We might conclude, therefore, that if an insurer sells n policies to n individuals, it assumes the total risk of the n individuals. In fact, the insurer, through careful underwriting and selection will end up with an average risk that is relatively smaller compared to the original risk to individual policyholders.

The explanation of this surprising result is a principle called ***the law of large numbers***, which states that as the number of observations increases, the difference between the observed relative frequency of an event and the true underlying probability tends to zero. Similarly, the difference between the observed average severity of an event (the average size of a loss) and the expected severity tends to zero as the number of observations increases. So, accurate prediction of outcomes is much easier with many separate (independent) risks than with only one or two.

Here is another way to see the reduced variability of outcomes based on larger samples. At a certain age, the probability of death within one year is .0010, or 10 in 10,000. If we have a sample of 10,000 lives, we can predict with 95% probability that the number of deaths will be between 4 and 16, a range of ±6 away from the mean of 10. If we have a sample of 1,000,000 lives, the 95% confidence interval is (938, 1062), a range of ±62 away from the mean of 1000. But we observe that the variability is 60% of the mean in the first case, but only 6.2% of the mean in the case with the larger sample.

As long as the individuals being insured are independent risks (i.e., a claim from one policyholder does not increase the probability of a claim from any other policyholder), then the larger the sample size, the smaller the variance of the average claim, and, hence, the smaller the risk. Thus, through the insurance mechanism, individuals can transfer their risks to an

insurer without having the insurer taking on an unmanageable level of risk in total.

In life insurance, the risk is associated with the variability in the number of death claims, which is modeled by a probability frequency distribution. In most property/casualty lines of insurance (e.g., auto), not only is there a frequency distribution for number of claims, but there is also a severity (or loss) distribution for size of claim, from which variability also arises. That is, given that a claim has occurred, the size of the loss payment is still highly variable.

By buying insurance, the individual policyholder transfers his or her risk to the insurer, but, because of the law of large numbers, the insurer ends up with a total risk that is manageable. This is illustrated in Figures 1.1a and 1.1b, showing the risk profiles for the individual and the insurer, respectively.

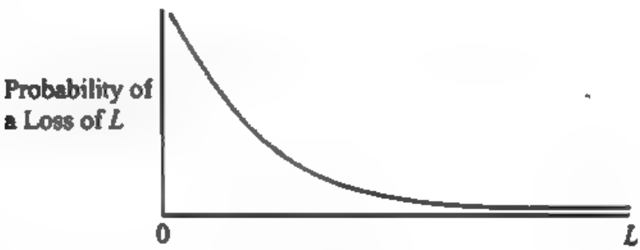


Figure 1.1a

For the individual, the probability is very high that there will be no loss at all from the defined event, but there is a non-zero probability of a significant loss. We denote the expected value of the loss to the policyholder by μ_{ph}, and the variance of the loss to the policyholder by σ^2_{ph}.

If the insurer selects n identical and independent policyholders, each with the same risk profile as that illustrated in Figure 1.1a, then the loss distribution for the insurer can be illustrated by Figure 1.1b.

For the insurer, the probability of no loss at all, given n policyholders, will be virtually zero if n is large, and the range of possible losses per policy is much smaller than for the individual policyholder.

Figure 1.1b

If the insurer selects n identical and independent policyholders, the expected value of the average loss per policy is μ_{ph}, the same as for the individual policyholder, but the variance of the average loss per policy is

$$\frac{\sigma_{ph}^2}{n},$$

or, equivalently, a standard deviation of

$$\frac{\sigma_{ph}}{\sqrt{n}}.$$

These results are derived in the following example.

EXAMPLE 1.1

Given n independent policyholders with individual loss random variables $X_1, X_2, \ldots, X_n$, such that the expected value of any policyholder's loss is μ_{ph} and the variance is σ_{ph}^2, show that for the insurer providing these n policyholders with insurance, the expected value of the insurer's average loss per policy is μ_{ph}, and the variance of the average loss per policy is $\frac{\sigma_{ph}^2}{n}$.

Solution

Let $S_n = X_1 + X_2 + \cdots + X_n$.

Let,

$$\bar{X} = \frac{1}{n} \cdot S_n = \frac{1}{n}(X_1 + X_2 + \cdots + X_n).$$

Then

$$E[\bar{X}] = \frac{1}{n} \cdot E[S_n] = \frac{1}{n} \cdot n\mu_{ph} = \mu_{ph},$$

and

$$Var(S_n) = Var(X_1 + X_2 + \cdots + X_n) = n \cdot \sigma^2_{ph}.$$

But

$$\begin{aligned} Var(\bar{X}) &= Var\left(\frac{1}{n} \cdot S_n\right) \\ &= \frac{1}{n^2} \cdot Var(S_n) \\ &= \frac{1}{n^2} \cdot n\sigma^2_{ph} \\ &= \frac{\sigma^2_{ph}}{n}. \end{aligned}$$

Hence we can see that the risk to the insurer, measured by the variance of the average loss, is only $\frac{1}{n}^{th}$ of the risk to the individual policyholder. □

1.3 INSURANCE AND UTILITY

It should be clear that the existence of a private insurance industry, of and by itself, will not decrease claim frequencies or loss severities. Viewed another way, merely by entering an insurance contract a person's expectation of loss does not change. Thus, with perfect information, the net premium for any policyholder would have to be the expected value of loss. But the policyholder would have to pay a gross premium in excess of the net premium so as to cover the expenses of selling and servicing the contract.

Why would someone pay a gross premium for an insurance contract that must exceed the expected value of the loss? The answer lies in a principle called the ***decreasing marginal utility of money***. According to this

principle, as extra units of wealth or income are added, the utility derived from such units decreases. This is displayed in the graphs that follow.

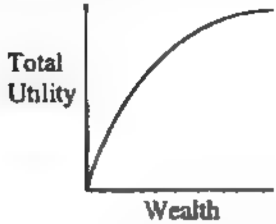


Figure 1.2a

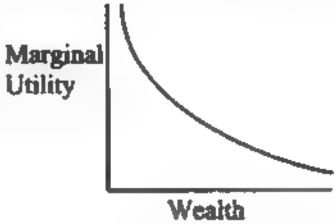


Figure 1.2b

As an example, with early dollars of income we buy food, clothing, and shelter, which represent high utility. With later dollars of income, we buy items such as a stereo for the jacuzzi, which is of lower utility.

The principle of decreasing marginal utility of money applies to anyone who is a ***risk averse,*** which is the case for most people. There are some people who are ***risk seekers***, for whom the principle of decreasing marginal utility does not apply. Such a person, for example, could be expected to forgo basic needs, such as food or shelter, to gamble on a chance for large wealth. The examples that follow assume that the purchaser of insurance is a risk avoider.

EXAMPLE 1.2

A prospective purchaser of insurance has 100 units of wealth. He faces a situation whereby he could incur a loss of Y units, where Y is a random loss with a uniform distribution between 0 and 36. This person has a personal utility curve given by $u(x)=\sqrt{x}$. What maximum gross premium would this person be willing to pay for insurance?

Solution

Note that for this individual $u'(x)>0$, so that u increases with x, and $u''(x)<0$, so that each additional unit of x brings less than one additional unit of utility, u. Hence this prospective policyholder is a risk avoider, since the law of decreasing marginal utility applies. (A risk seeker would have an increasing marginal utility curve.)

Further, noting that the p.d.f. for the random loss is $f(y)=\frac{1}{36}$, we can find

$$\begin{aligned} E[Y] &= \int y \cdot f(y)\ dy \\ &= \int_0^{36} \frac{y}{36}\ dy \\ &= \left.\frac{y^2}{72}\right|_0^{36} \\ &= 18, \end{aligned}$$

so the expected value of the loss is 18. The insurer must therefore charge a gross premium in excess of 18 to cover sales commissions and administration costs.

Why would a policyholder pay more than 18 to buy insurance whose expected value is 18? The answer lies in the marginal utility curve for this policyholder illustrated in the following figure.

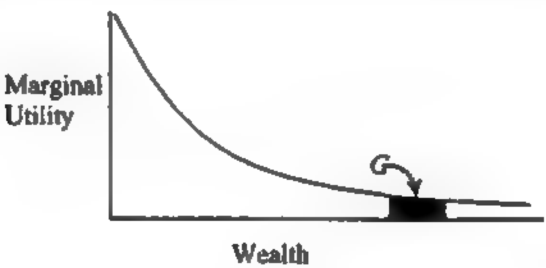


Figure 1.3

The policyholder will pay a gross premium of G for the insurance, so he loses G whether or not the loss occurs, leaving him with $100-G$ units of wealth. Without insurance, however, the policyholder faces a possible loss of 36 units of wealth, which is 36% of his total wealth.

If the policyholder buys insurance, the resulting wealth position is certain; it will be $100-G$, with utility value $\sqrt{100-G}$. If he does

not buy insurance, the resulting wealth position is probabilistic, given by $100-Y$, and the expected utility value of the resulting wealth position can be calculated as

$$\begin{aligned} E[U] &= \int_0^{36} u(100-y)\cdot f(y)\ dy \\ &= \int_0^{36} \sqrt{100-y}\cdot\frac{1}{36}\ dy \\ &= \frac{1}{36}\left\{-\frac{2}{3}(100-y)^{3/2}\right\}\Big|_0^{36} \\ &= \frac{244}{27}. \end{aligned}$$

The policyholder should be willing to pay a premium G that equates the expected utility values of the resulting wealth positions with or without insurance. Thus we find G such that $\sqrt{100-G}=\frac{244}{27}$, which results in $G=18.33$. Thus the policyholder will pay up to 18.33 for this insurance, which exceeds its expected value of 18, and if the insurer can charge a premium less than 18.33, the insurance purchase will be made. □

Given this or a similar utility function, we can see why it may not make sense to insure against small losses (e.g., theft of goods worth less than \$200). In this case, the utility value of the gross premium will exceed the expected utility value, because we have not moved far enough in the decreasing marginal utility curve to overcome the expense element inherent in the gross premium.

EXAMPLE 1.3

You are trying to decide whether to invest in Company A or B. For this investment, the utility profile can be measured by the function

$$u(P) = \sqrt{P-100},\ P>100,$$

where P represents profit.

(a) Show that this is the utility function of a risk avoider.
(b) Given the following information, determine your investment strategy based on (i) expected monetary value, and (ii) expected utility value.

		Profit	
	Probability	Company A	Company B
Economy Advances	.40	4000	2800
Economy Stagnates	.60	200	400

Solution

(a) Given that

$$u(P) = \sqrt{P-100},$$

then

$$u'(P) = \frac{1}{2}(P-100)^{-1/2}$$

and

$$u''(P) = -\frac{1}{4}(P-100)^{-3/2}.$$

This shows that

$$u'(P) > 0, \quad \text{for} \quad P > 100,$$

and

$$u''(P) < 0, \quad \text{for} \quad P > 100,$$

so the investor is risk averse.

(b) The following table shows the monetary payoffs and their associated utilities.

		Profit	
	Probability	Company A	Company B
Economy Advances	.400	4000(62.45)	2800(51.96)
Economy Stagnates	.600	200(10.00)	400(17.32)

(i) Expected monetary value:

E(Company A) = .40(4000) + .60(200) = 1720

E(Company B) = .40(2800) + .60(400) = 1360

Invest in Company A.

(ii) Expected utility value:

$E(\text{Company A}) = .40(62.45)+.60(10.00) = 30.98$

$E(\text{Company B}) = .40(51.96)+.60(17.32) = 31.18$

Invest in Company B. □

EXAMPLE 1.4

An individual faces the following possible losses:

Loss Size	Probability
$1000	0.001
100	0.100
0	0.899

If the utility function of a potential purchaser of insurance is:

$$u(x) = x^{0.6}$$

(a) Show that this person is risk averse.

(b) Calculate the maximum premium this individual would pay for insurance given the above loss distribution and initial wealth of $2000.

Solution

(a)
$$\begin{aligned} u(x) &= x^{0.6} && > 0 \text{ if } x>0 \\ u'(x) &= 0.6x^{-0.4} && > 0 \text{ if } x>0 \\ u''(x) &= -0.24x^{-1.4} && < 0 \end{aligned}$$

So we have decreasing marginal utility, which indicates the individual is risk averse.

(b) With insurance that costs $\$G$, the outcome is known and equals $2000-G$ with utility $(2000-G)^{0.6}$.

Without insurance, we have a loss distribution with three possible outcomes and resulting expected utility.

$$\begin{aligned} &.001(\$1000)^{0.6}+.100(\$1900)^{0.6}+.899(\$2000)^{0.6} \\ &\qquad = 0.63095734+9.273681167+85.97608973 \\ &\qquad = 95.31286663 \end{aligned}$$

$$\text{So set } (2000-G)^{0.6} = 95.3128663$$
$$G = \$11.22$$
$$\text{Note: } E[L] = .001(\$1000)+.100(100) = \$11.00$$
$$\text{So } G > E[L].$$ □

1.4 What Makes a Risk Insurable

We have shown in the previous sections that an individual will see the purchase of insurance as economically advantageous if the principle of decreasing marginal utility applies (i.e., the individual is a risk avoider). On the other hand, the insurer will agree to insure a prospective policyholder if the law of large numbers can be applied to the risk pool to which the prospective policyholder wishes to belong. With these principles in mind, what makes a risk insurable?

(1) ***It should be economically feasible.*** If we do not move far enough on the utility function, then the utility gained by insuring will not be enough to cover the utility of the cost of the insurance mechanism (e.g., sales commissions and head office expenses).

(2) ***The economic value of the insurance should be calculable.*** An example of where this criterion holds is auto collision insurance. Here a large number of small losses are experienced. We can get a lot of data on collision experience and, through the law of large numbers, can calculate an expected premium with a high degree of confidence. Insuring a nuclear reactor against meltdown is an example of where this criterion does not hold. Such a policy can be issued by using a risk-sharing arrangement among many insurers so that the exposure to risk for any one company is manageable.

(3) ***The loss must be definite.*** This criterion is meant to guard against policyholder manipulation and moral hazard. Moral hazard occurs when the insured is able to increase the value of the insurance beyond that expected in the price or premium. A car accident with police documentation is definite. Death is definite. What is not so definite, but still insured, is disability. When is an insured well enough to return to work? How do you guard against malingering?

(4) ***The loss must be random in nature.*** Again we wish to have the insured event beyond the control of the policyholder. The presence of criteria three and four allow the actuary to assume random sampling in

the projections of future claim activity. That is, there is no statistical bias in the selection of one insurance unit versus another.

(5) ***The exposures in any rate class must be homogeneous.*** This means that, before the fact, the loss expectation for any unit in a class must be the same as for any other unit in the class. In terms of random sampling, this is analogous to each elementary unit having the same probability of being drawn. Through anti-selection by policyholders, this criterion might not be satisfied. **Anti-selection** occurs when the policyholder has more information than the insurer, and the policyholder uses that extra information to gain a price/rate loss advantage.

(6) ***Exposure units should be spatially and temporally independent.*** In terms of random sampling, this implies that selection of one elementary unit does not affect the probability of drawing any other elementary unit. In more practical terms, we wish to avoid any catastrophic exposure to risk. We would not, for example, insure all the stores in one retail area, since one fire or one riot could result in a huge loss. In insurance terms, the fact that one insured has a claim should not affect whether another insured has a claim.

These criteria, if fully satisfied, mean that the risk is definitely insurable. The questions of risk classification and price still follow. On the other hand, the fact that a potential risk exposure does not fully satisfy the criteria does not necessarily mean that insurance will not be issued. Some special care or risk sharing in these circumstances (e.g., reinsurance) may be necessary. In property/casualty insurance, rarely does an insurable risk meet all of the listed criteria.

1.5 WHAT INSURANCE IS AND IS NOT

There is often confusion in the minds of consumers and regulators as to the purposes and intent of insurance.

The insurance mechanism is used to transfer risk from the individual policyholder to the pooled group of policyholders represented by the insurance corporation. If the insured pool is a large collection of independent policyholders then the per-unit risk will be greatly reduced and will be manageable for the insurance company. The insurance company administers the plan, invests all funds, pays all benefits, and so on. The insurance company can only pay out money that comes from the pooled funds. If claims rise, so too must premiums.

From the policyholders' viewpoint, ***insurance*** is available only for pure risks; that is, where the outcome is either loss or no loss. The policyholder cannot profit from buying insurance.

In ***speculation***, there is also a transfer of risk, in that an individual can transfer an unwanted risk to a speculator. The motive for the speculator is the chance to make a profit.

A good example of how speculation can be used to transfer risk is the futures market. Suppose a farmer plants a field of winter wheat in October. He will deliver this wheat in July. This farmer is risk averse and does not wish to speculate on what the price of grain might be in July. The farmer goes to the futures exchange and sees that it is possible to sell the grain in October to a speculator for $4 a bushel with delivery in July. In July, grain is actually selling for $3.50 a bushel. The farmer delivers the grain as agreed and is paid $4 a bushel. The speculator must now realize the loss of $0.50 a bushel. Had grain prices risen to $4.75 a bushel (e.g., in a dry summer) the speculator would have made a profit of $0.75 a bushel.

By taking on this risk, the speculator does two positive things. First, the risk of fluctuating prices is removed from the risk averse farmer and assumed by the speculator (who hopes to make a profit). To the extent that the speculator is correct in his/her projections, prices are stabilized. Note, however, that the risk has only been transferred; it has not been reduced or removed.

There are two key differences between speculation and insurance. The first is the profit motive behind speculation. There is no profit motive on the part of the policyholder in entering an insurance agreement (the insurer, however, hopes to make a profit). Second, the insurance process significantly reduces total risk through the Law of Large Numbers. Speculation transfers risk, but does not reduce it.

In ***gambling***, risk is created where none existed and none needed to exist. In terms of utility, gambling works in a fashion opposite of insurance. People spend early and high utility dollars in the hopes of gaining large wealth that has lower utility value. Overall, gambling decreases societal utility by redistributing income in a non-optimal fashion. Some theorize that gamblers have utility curves that explain their actions, i.e., both $u'(x)$ and $u''(x)$ would be positive.

If the profits from the gambling process (e.g., a state or provincial lottery) are spent on high utility needs (e.g., a hospital), then it is possible for the final result of this process to increase total societal utility. Otherwise gambling decreases total utility and is a waste of human resources.

1.6 Risk, Peril, and Hazard

Risk is a measure of possible variation of economic outcomes. It is measured by the variation between the actual outcome and the expected outcome.

Peril is used as an identifier of a cause of risk. Examples include fire, collision, theft, earthquake, wind, illness, and so on.

The various contributing factors to the peril are called ***hazards***. There are physical hazards such as location, structure, and poor wiring, and there are moral hazards such as dishonesty, negligence, carelessness, indifference, and so on.

An example might help. Mr. Rich owns a cabin cruiser. Hazards when sailing are negligence on the part of the captain, rocks, shoals, and so on. These are contributing factors. Perils would be things like fire or collision (i.e., cause of risk) which may or may not cause a financial loss, which is risk.

In conclusion, an insurance contract will reimburse the policyholder for economic loss caused by a peril covered in the policy. Thus the policyholder transfers this risk to the insurance company.

1.7 Purchase of Insurance: Other Reasons

While utility theory provides an underlying economic rationale for the decision to purchase insurance, quite often some other practical reasons are present:

(1) Legal requirements. Most jurisdictions have financial responsibility laws that apply to all licensed motor vehicles. The licensee must show that he or she can satisfy judgments rendered as a result of accidents resulting from operation of the vehicle. The most popular way of satisfying this requirement is through insurance. There are other laws and regulations that require insurance before a license to engage in certain businesses is issued.

(2) Lenders' requirements. When an individual takes out a mortgage on property or takes out a loan to purchase a vehicle, the lender almost always requires insurance on the property or vehicle up to the amount of the loan (this to protect the lender's insurable interest in the property). This is also common for commercial loans, which are secured by property.

(3) Commercial requirements. In the course of business transactions, one party will often obligate itself in some measure to perform a service, to deliver a product, etc. It is common that insurance is purchased to compensate the injured party if the service is not performed or the goods are not delivered. Such business arrangements are often contingent on the performing party obtaining insurance.

(4) Special expertise. The insurance company may provide a service on a more cost-effective basis than the insured can do on its own. The most obvious example is adjustment of claims. Insurers have large, experienced, claim departments. An example of this would be using an insurance company to administer the paperwork of a large dental insurance program. Some companies also see value in having a "third party," the insurer, handle claims made by its customers. Other services include boiler inspections, and loss control audits.

(5) Taxation. If a company in the United States or Canada self-insures its exposures, it can only claim a tax deduction for losses as they are paid. In contrast, the cost of insurance is expensed immediately since the premium is paid up front. Thus in "long-tailed" lines such as product liability, the deduction for income tax purposes can be accelerated by many years and provide a real economic benefit.

1.8 Exercises

Section 1.2

1.1 (a) State the law of large numbers.
    (b) Explain the importance of the law of large numbers to the insurance mechanism.

Section 1.3

1.2 Confirm that the utility function log, for $u(x) = k \cdot \log x$, and $k > 0$ $x > 0$, is the utility function of a decision maker who is risk averse.

1.3 Which of the following two proposals in the table below would a risk avoider choose?

	Proposal A		Proba-	Proposal B		Proba-
Outcome	Payoff	Utility	bility	Payoff	Utility	bility
O_1	80,000	1.0	.6	50,000	.9	.5
O_2	10,000	0.5	.1	30,000	.8	.3
O_3	−30,000	0.0	.3	−10,000	.2	.2

1.4 Two businessmen view the following proposals.

	X		Y	
	Success	Failure	Success	Failure
Profit	50,000	−20,000	5,000	−5,000
Probability	.35	.65	.55	.45

Their respective utility schedules for the project are as follows.

	Businessman	
x	A	B
− 20,000	.300	.550
− 5,000	.450	.709
+ 5,000	.550	.770
+ 50,000	1.000	1.000

What decisions would they make based on:
(a) expected monetary value, and
(b) expected utility value?

1.5 Assume the management of an investment firm has utility function, for any project, $U(P) = \sqrt{P-1000}$, where P represents profit.

(a) Confirm that management is risk averse.
(b) Consider the following two proposals, below:

Proposal A	
Profit	Probability
3000	.10
3500	.20
4000	.40
4500	.20
5000	.10

Proposal B	
Profit	Probability
2000	.10
3000	.25
4000	.30
5000	.25
6000	.10

Which proposal would management choose based on:
(i) expected monetary value, and
(ii) expected utility value?

1.6 A market gardener faces the possibility of an early frost that would destroy part of his crop. He can buy crop insurance. This creates four possible outcomes, which are presented, in the following table.

	Profit	
	Freeze	No Freeze
No Insurance	10,000	30,000
Insurance	20,000	25,000

(a) Based on expected monetary value, what probability must the farmer attach to early frost to make buying insurance a wise decision?

(b) Given his existing wealth, the farmer has the following utility profile.

Profit	Utility
10,000	71
20,000	123
25,000	141
30,000	158

Based on expected utility value, what probability must the farmer attach to an early frost to make buying insurance a wise decision?

1.7 You are subject to the utility function $u(x)=\left(\frac{x}{10,000}\right)^{.9}$, where x is wealth. Your current wealth is 50,000. What is the maximum premium you would pay to insure against a loss that is uniformly distributed between 0 and 30,000?

1.8 You follow the utility function $u(x)=1-\exp\left(\frac{-x}{100,000}\right)$, where x is wealth. Your current wealth is 20,000. What is the maximum amount you would pay to take part in a fair coin toss where you have .5 probability of winning 10,000? If you win you do not receive a return of your wager.

1.9 A person has a utility function, over the relevant range, given by $u(x)=10,000x-x^2$, where x is wealth. Her current wealth is 3000. What is the maximum wager she would make in a game where there is a 30% chance of winning 2000 plus the return of her wager?

1.10 You are given the following information.

(i) The gross premium for insurance is 4500.

(ii) The individual knows he will have 1, 2, or 3 losses with equal probability.

(iii) Each loss will cost 2000.

(iv) $u = \mu+\sigma/6$ measures the loss of utility for the individual, where u is a measure of utility, μ is the expected value of loss, and σ is the standard deviation of loss.

Under these conditions, determine whether the prospective policyholder will buy insurance. Why?

1.11. Mr. Smith has a total wealth of 525,000 and his utility of wealth is $u(x) = \ln(x)$. He owns a sports car worth 50,000. The insurance on his sports car is due for renewal. Based on Mr. Smith's driving record, the risk of damage to his car in the next year is as follows.

Amount of Damage	Probability
0	.80
10,000	.15
20,000	.04
50,000	.01

Mr. Smith's insurance company charges premiums for all its policies equal to the expected value of its claim payments under the policy plus 10% of this expected value as a loading.

(a) Should Mr. Smith fully insure his car at the insurance company's premium? Explain why or why not.

(b) As an alternative to its full coverage policy, the insurance company is offering a new policy that will pay 50% of all damage amounts for accidents greater than or equal to 20,000. All other damage amounts are paid by the insured. Should Mr. Smith insure his car with this new policy?

Section 1.4

1.12 It is common for successful race horses to be sold for stud (breeding purposes) at the end of their racing careers. Not all such horses are "successful." Should it be possible to buy insurance to indemnify you for loss if a race horse you buy is not a successful breeder?

1.13 The XYZ Insurance Company has been asked to issue a 2-year term insurance policy on a specially trained dog that is going to star in a movie. If the dog dies in year one, 8000 will be paid at the end of year one. If the dog dies in year two, 5000 will be paid at the end of year two. If the dog lives to the start of year three, no payment is made and the contract ends. The dog is now age x, and the insurance company develops the following survivorship data based on known mortality experience of dogs of the given age and breed.

$$\begin{aligned}\ell_x &= 7000\\ \ell_{x+1} &= 6000\\ \ell_{x+2} &= 4500\\ \ell_{x+3} &= 2500\\ \ell_{x+4} &= 0\end{aligned}$$

(a) Is this an insurable risk?

(b) If $i = 10\%$, determine the net single premium for the contract.

(c) Calculate the associated variance.

Section 1.5

1.14 From an economic viewpoint, compare and contrast gambling and insurance. Briefly explain why insurance is more acceptable.

Section 1.6

1.15 (a) Differentiate among risk, peril, and hazard.

(b) Give an example of each.

PROPERTY/CASUALTY COVERAGES ○ 2

2.1 INTRODUCTION

Chapter One outlined the reasons that consumers buy insurance. In Chapter Two we will review some of the most widely sold coverages available from property/casualty insurance companies and the areas of economic insecurity for which the property/casualty insurance industry provides insurance. We will describe these coverages in a very general and generic way. That is, it is not the purpose of this chapter to outline any particular contract used by any particular insurer. Rather, we will attempt to describe property/casualty coverages that exist around the world and the general level of security that they provide.

Readers are invited to review their own insurance coverage at this time. Most of you will have some property/casualty insurance contract, be it an automobile insurance policy or insurance for a homeowner or tenant. It will prove advantageous to look at your particular coverage when you are reading the generic description of that coverage in this chapter.

2.2 AUTOMOBILE INSURANCE

Important coverages normally available in an automobile insurance policy include:

- ***liability insurance***
- ***medical benefits***
- ***uninsured and underinsured motorist coverage***
- ***collision insurance***
- ***other than collision (OTC) insurance***

In many jurisdictions, the first two coverages are compulsory in that the law requires each auto owner to purchase insurance that meets the ***financial responsibility limits***, whereas the others are usually purchased at the option of the policyholder. If you require a loan to finance the

purchase of your car, then the financial institution may require collision and comprehensive coverage.

The insured parties are the named policyholder and immediate family. The policy coverage applies when the policyholder or family member is driving one of the vehicles listed in the policy declarations (your covered auto). Coverage also applies if the covered auto is being driven by an "invited driver" and coverage usually extends to a utility trailer attached to the vehicle. The policy does not cover normal operating expenses such as wear and tear, depreciation, rust, and so on. Coverage normally ceases if the vehicle is being used for commercial purposes. ***Commercial auto policies*** exist for this purpose, but with a different price structure. They will not be discussed further in this text.

Legislative reforms over the past decade have significantly changed the importance of Liability Insurance (Section A of the policy) versus Medical Benefits (Section B of the policy).

We now generally have two types of legislation defining how the auto insurance benefits will be determined: at fault and no-fault. In the United States, 38 states require that the injured party prove that the insured was ***at fault*** in the accident and is therefore liable for the payment of claim to the injured party.

Benefits under the liability section of the auto policy are available only if the liability insurer believes its insured was at fault in the accident or the injured party sues the insured and proves that the insured was at fault in the accident. This may require a lengthy court case, although the majority of cases are settled out of court. This at-fault system of settlement is also called the tort system. Evidence exists that in the ***tort*** system small claims are highly overcompensated, usually through out-of-court settlements, whereas larger claims are compensated for as little as 30% of their costs. Further, only about 25% of the premium dollar ends up in the hands of the injured party. The other 75% is consumed by legal fees, court costs, and insurer administration expenses. Because of this, many jurisdictions have implemented no-fault auto insurance systems.

Under a ***no-fault*** system, the injured party does not have to sue for compensation or even prove that the driver of the other car was at fault for the accident. Instead, the benefits that would normally be paid by an at-fault party's liability insurance become payable under the insured's

personal injury protection. Thus, instead of the injured party suing for damages because of bodily injury or property damage, the injured party gets the level of benefits defined in the policy's personal injury protection policy (accident benefits in Canada). Thus the benefits are not paid by the insurer of the at-fault driver; the benefits are paid by the injured party's own insurer. The tort system liability premium is significantly decreased while the personal injury protection premium is significantly increased under a no-fault system. In theory, the total premium under the no-fault system should be lower than the total premium under the tort system because many legal and court expenses are reduced or eliminated. On the other hand, the flexibility of the tort system is lost to the defined benefits of the no-fault system.

At the time of writing, all provinces in Canada and twelve states in the United States had some kind of no-fault legislation. There are virtually as many no-fault insurance systems as there are jurisdictions with these systems. For example, some jurisdictions use a system called ***threshold no-fault***, under which you look first to your own insurer for defined no-fault benefits for any minor accident, but if your injuries exceed a defined threshold (e.g., death, total and permanent disability, disfigurement, or medical expenses which exceed a specified amount), you can then pursue a tort settlement. In this case, the court may have to make a two-part decision. First, does your injury exceed the defined threshold? Second, if it does, what is the appropriate tort settlement given that fault has been established?

In three other provinces in Canada, the provincial government runs the auto insurance system through a ***government monopoly***. In these provinces, there is no issue as to who is at fault since there is only one insurer/payor. Government monopolies for auto insurance are rare around the world. Government *regulation* of coverage and/or rates is, however, fairly common. Even in a pure no-fault environment, the police will still be asked which driver was at fault or the degrees to which the drivers shared in the fault, because, in most jurisdictions, at-fault events cause the premium to rise at the next policy renewal, and remain elevated for several years.

Virtually all auto policies provide coverage for out-of-state or out-of-province accidents, up to the amounts required in the jurisdiction where the accident occurs. Most policies issued in Canada or the United States limit liability coverage to just Canada and the United States.

2.2.1 Liability Insurance (Section A)

This coverage is commonly referred to as Third Party Liability, or Section A, or BI/PD (which stands for Bodily-Injury/Property Damage).

The ***liability*** section of the auto policy provides coverage to the policyholder if, as the driver of a covered vehicle, the policyholder injures a third party or damages a third party's property. The policyholder's property is not covered by the auto liability insurance, but would normally be covered under the policyholder's homeowners policy. Although most liability incidents are settled without going to court, in a tort jurisdiction (i.e., at-fault), if the policyholder is sued with respect to negligence for such bodily injury or property damage, the insurer will provide legal defense for the policyholder and, if the policyholder is found to be liable, the insurer will also pay, on behalf of the policyholder, damages assessed against the policyholder up to the limits of coverage defined in the policy. Note that the total cost of legal defense and payment of damages can exceed the policy limits because only the payment for damages is subject to the policy limits. The insurer is allowed, however, to cease its legal defense when the amount it has paid for damages reaches the policy limits. A person who intentionally causes a loss is not covered. There is also no coverage if workers compensation is supposed to provide the benefits.

Third-party liability coverage is compulsory in virtually all jurisdictions before you can legally drive your car (although many drivers drive illegally without such insurance). Virtually all jurisdictions specify some minimum level of liability coverage, such as $25,000 in most states and $200,000 in most provinces in Canada. Higher limits are available for an extra premium. There may be separate (called ***split***) limits for the bodily injury and the property damage coverages (the latter limit is normally smaller). Further, the bodily injury coverage may specify a limit per person injured per occurrence and a second overall total limit per occurrence (e.g., $50,000 per person and $100,000 per occurrence). Because bodily injuries can result in claims of millions of dollars and the courts can attach your personal assets if the assessed damages exceed your insurance coverage, you are well advised to buy increased limits of liability coverage.

Premiums for the liability coverage vary by the limits chosen, the territory in which the vehicle is garaged, the use of the automobile (e.g., pleasure or business), the accident claims and moving vehicle convictions record of the policyholder, and often the age, sex and marital status of the policyholder if not precluded by human rights legislation or regulatory rules.

2.2.2 Medical Benefits (Section B)

Generally, the second policy coverage (Section B) is called ***medical payments*** (in a tort jurisdiction), ***personal injury protection*** (in a no-fault jurisdiction), or ***accident benefits*** (in Canada). This coverage provides protection to the policyholder and family in the case of injury in an accident for which the policyholder would be liable (you cannot sue yourself for coverage under third party liability, so this is the alternative), or for any accident in a no-fault jurisdiction. Again, this coverage is usually compulsory.

Personal injury protection provides defined levels of benefits to the injured policyholder or family member for income replacement, medical care, rehabilitation, home care, survivor's benefits, and so on. This is an example of *first*-party coverage since the policyholder receives the claim payments. Generally there is no coverage if workers compensation is supposed to provide the benefits.

In a tort (at-fault) jurisdiction, the importance of Section A coverage (and its costs) exceed the importance of Section B. In a no-fault jurisdiction, the reverse is true.

2.2.3 Uninsured and Underinsured Motorist Coverage

A third section of the policy usually provides protection for the policyholder and family if injured by either an ***unidentified***, ***uninsured***, or ***underinsured*** motorist (i.e., someone who is insured but at lower liability limits than purchased by the policyholder or required by law). Under this coverage, in a tort jurisdiction, the policyholder has coverage from his or her own insurer equivalent to what would have existed had the motorist causing the accident been identifiable or fully insured. In this respect, this coverage is similar to no-fault where your insurer covers your injuries and damages (even if the uninsured or underinsured driver was at fault). Note that this benefit provides an incentive for the policyholder to buy larger liability limits of coverage for himself or herself.

2.2.4 Collision And Other Than Collision (Section C)

A fourth section of a typical auto insurance policy provides coverage for damage to the policyholder's own vehicle, under two subsections: one covering ***collision*** and another covering ***other than collision*** (OTC). The policyholder has the option to purchase one or the other, or both, of these coverages.

Under collision insurance, if your vehicle is damaged in an accident, the insurer will pay the cost of its repair or replacement as defined in the policy, normally subject to a ***deductible*** such as $500. This means that the policyholder is responsible for the first $500 of the repair or replacement cost. This tends to eliminate the filing of small claims for which the cost of administration and settlement would likely exceed the benefit. It also provides an economic incentive for the policyholder to prevent accidents, since the policyholder now carries some risk.

The ***collision limit*** is the lesser of the actual cash value of the damaged property or the amount necessary to repair or replace it. The insurer reserves the right to pay for the loss in money or repair or replace the damaged property. As an aside, on some new cars the depreciated value of the car (which is the policy limit) may be less than the outstanding balance of the car loan, which can create potential problems in the case of a serious accident. Special provisions exist for such instances.

If another driver is at fault for the accident in a tort jurisdiction, the insurer that paid the policyholder the defined collision benefits can sue the at-fault driver and recover its costs and the deductible from the at-fault driver or his or her insurer. If the suit is successful, the amount of the deductible (now recovered) will be paid to the policyholder. If the insurer decides not to sue, the policyholder can try to recover the deductible from the alleged at-fault driver or insurer, but at the policyholder's risk and expense.

The ability of the insurer to sue the at-fault driver in a tort jurisdiction and recover its costs is called ***subrogation***. Technically, subrogation means that the insurer, once it has indemnified the policyholder, automatically assumes the legal rights of the policyholder to sue. As a result of the subrogation process, premiums for collision insurance tend to be lower, whereas premiums for liability insurance tend to be higher than without subrogation. However, the resulting liability and collision premiums are more appropriate and equitable because the resultant premiums reflect the true costs that are brought to the insurance pool by the policyholder. Also the at-fault party, rather than an innocent victim, bears the cost. Subrogation is also important because the liability coverage is normally compulsory whereas the collision coverage is not.

Another legal right of the insurer under an auto insurance policy is called ***salvage***. If a collision claim requires paying the full value of the vehicle

(known colloquially as a write-off), the ownership rights to any remaining value in the vehicle accrue to the insurer. Thus, the insurer has the right to take the vehicle to a wrecker and retain any salvage value that may remain. If the salvage value exceeds the amount the insurer originally paid to the policyholder, the payment to the policyholder must be increased to at least the salvage value payment. That is, the insurer cannot profit from the salvage provision. The salvage provision ultimately decreases the premium required for the collision coverage.

Collision premiums vary according to the type of vehicle (based on its value plus an index of damageability and cost to repair), its use (e.g., business or pleasure), and normally also the territory in which the vehicle is garaged. Almost all jurisdictions allow premiums to vary according to the accident history of the driver. Finally, most jurisdictions allow premiums to vary based on the policyholder's age, gender and (sometimes) marital status. The use of these parameters has become a human rights issue in some jurisdictions. Statistics do exist that clearly show correlation between each of these risk classification factors and the expected costs brought to the risk class by the policyholder.

The final optional subsection of a typical auto insurance policy covers perils other than collision (OTC), such as hail, fire, vandalism, stone chips, theft, and so on. Excluded perils include war, acts of terrorism, damage due to wear and tear, road damage to tires, radioactive contamination, damage due to the discharge of a nuclear weapon, and, of course, collision.

If coverage is provided for all perils except those specifically excluded, such as those just listed above, then it is referred to as ***comprehensive*** coverage. If, however, the policy only covers perils that are specifically listed (as opposed to all perils except those specifically excluded), then the coverage is referred to as ***specified perils***. OTC premiums usually vary only by the type of vehicle (its value and expectation as to ease of damage and cost to repair), and the territory where the vehicle is garaged. Personal attributes of the policyholder (e.g., age, sex, marital status, claims history) are not normally deemed to be relevant in pricing this coverage.

It is possible to combine the collision and OTC coverages into an ***all risks*** coverage, but this is seldom done. In practice, a policyholder's collision deductible is normally significantly higher than the OTC

deductible (e.g., $1,000 versus $500). Policyholders often choose to lower their collision and OTC coverage as the vehicles age because the value of the vehicles decrease. Antique car owners are an exception to this as the vehicles increase in value with age. Insurers offer antique car coverage usually for an 'agreed-upon value' for the car.

2.3 Homeowners Insurance

As with auto insurance, the typical ***homeowners insurance*** policy has different sections that specify different insurance coverages. Section I of the policy is normally subdivided into four subsections often called Coverages A, B, C, and D.

Coverage A provides protection against damage to your house (dwelling) on named perils or an all-risks basis up to a set limit. Earthquake and flood are generally excluded from standard coverage. These exclusions and limits are designed to make rates more affordable and equitable, given the expected costs that are brought to the insurance pool by the policyholders.

Loss or damage occurring after the dwelling has been vacant for more than thirty consecutive days and damage caused by nuclear accidents or acts of war or terrorism are not covered, nor are buildings used for business or farming purposes. Finally, loss or damage resulting from intentional or criminal acts of the insured is not covered.

The fact that not all perils are covered brings up the matter of determining exactly which peril did, in fact, cause the loss. Under an important principle called the ***doctrine of proximate cause***, a loss is covered only if a ***covered peril*** is the proximate cause of a ***covered consequence*** (note the need for both a covered cause and a covered consequence). A peril is covered if it is a named peril in a specified perils policy or it is not an excluded peril in a comprehensive or all-risks policy. A covered peril is the proximate cause if it initiates an unbroken sequence of events leading to a covered consequence. If, for example, a windstorm causes a power outage, and as a result of the power outage all of the food in your freezer goes bad, then this loss is covered if you are covered for wind (a covered peril) and for reimbursement of food spoilage (a covered consequence). Wind is the proximate cause.

Homeowners dwelling coverage comes with a deductible to preclude small claims which are both administratively expensive and, given the utility concepts of Chapter One, uneconomical to the insured. It also puts the policyholder at risk (for the deductible) which should result in more responsible actions by the insured.

In the previous section on auto insurance, we described subrogation, which also exists in any homeowners policy. To show how subrogation might work with homeowners dwelling insurance, consider the following scenario.

A homeowner experiences a \$50,000 fire in the home. It is established that the cause of the fire is faulty wiring in a kitchen appliance. When the insurer pays the homeowner the \$50,000 necessary to rebuild the kitchen, it acquires the legal rights of the homeowner to sue the appliance manufacturer for negligence. If successful, the insurer can recover its \$50,000 of costs. (If it recovers more, the extra amount would be paid to the homeowner.) In this instance, the \$50,000 was initially considered a homeowner's loss, but after it is recovered from the manufacturer's insurer it will be viewed instead as a product liability loss.

This scenario alludes to another feature that may exist within a homeowners policy. We noted earlier that there is a deductible within the homeowners policy to avoid the expense of small claims. This deductible may be defined so as to disappear after the loss reaches a certain size. This is illustrated more fully in the example that follows.

EXAMPLE 2.1

A homeowners dwelling policy has a deductible of 250 for claims up to 1000. Between 1000 and 2000, the deductible disappears linearly so that for claims of 2000 or more, there is no deductible. Determine the payment that would be made to the policyholder on a claim of 1300.

Solution

Set up a diagram to show the linear disappearance of the deductible.

Claim	1000	1300	2000
Deductible	250	X	0
Loss Payment	750	$1300-X$	2000

Using linear interpolation, $X=\left(\frac{2000-1300}{2000-1000}\right)(250)=175$, so the resulting loss payment is $1300-175 = 1125$. ❑

An interesting feature in the dwelling insurance coverage on a replacement cost basis is a provision called ***coinsurance***. Most homes experience little or no damage year after year (at least nothing in excess of the deductible). Even if there is a claim, it is usually far less than the full value of the home or the amount of insurance purchased on it. Thus, most property losses are partial, not total, and are relatively small. For homeowners insurance, the loss distribution is heavily weighted toward the smaller claim amounts.

What would happen, however, if a policyholder only purchased partial coverage? For example, what benefit or premium adjustment would be fair if a policyholder bought $200,000 of insurance on a $300,000 home, and then suffered a $40,000 loss? Clearly the premium for $200,000 of coverage should be more than two-thirds the premium for $300,000 of coverage on the same house. To avoid this problem, insurers require that policyholders insure their homes to near full value in order to get full coverage; otherwise the coinsurance clause is activated.

Normally, if the insurance equals at least 80% of the value of the house *at the time of the loss*, it is deemed to be insured for the full value. Demanding 100% coverage would be difficult because of the movement of real estate values from one policy anniversary to the next. That is if:

$$I(X) = \text{Insurance for Loss } X$$

Then:

$$I(X) = \min\left\{SI, \frac{SI}{.80 \cdot FV} \times loss\right\}$$

where SI = Sum Insured
FV = Full Value

An example of how this clause works may be helpful.

EXAMPLE 2.2

A homeowner has a house valued at 300,000 at the time of a loss event, but has insured it for 200,000 with an insurer that requires 80% of full coverage before it reimburses losses in full. If coverage is less than 80% of full coverage, then any loss is reimbursed on a pro-rata basis of what would have been paid had the 80% requirement been met. The homeowner has a kitchen fire estimated at 40,000 on a replacement cost basis. How much will the insurer pay toward reimbursing the homeowner for this loss?

Solution

For full coverage, the homeowner needed insurance equal to at least 240,000 (80% of 300,000) as measured at the time of loss. In this case the homeowner only had 200,000 of coverage. Thus, on the 40,000 kitchen fire, the insurer would pay

$$\frac{200,000}{(.80)(3,000,000)} \times 40,000 = 33,333.$$

The amount of reimbursement paid by the insurer is further limited by the policy limit of the policy (200,000 in this case).

Note: The value of the home used in the coinsurance clause is the value at the time of loss. □

In determining compliance with the coinsurance requirement, the insured is permitted to deduct the cost of excavations and pipes, wiring or foundations that are below the basement (or below the ground if there is no basement), from the replacement cost of the home.

There are several arguments in favor of the coinsurance clause, including the following:

(1) It encourages insurance to full value by penalizing under-insurance.

(2) The use of the coinsurance clause results in greater premium equity among insureds. If losses are skewed to smaller claims, but the premium on any given home is a linear function of the amount of insurance purchased, then, without a coinsurance clause, persons who purchased small amounts of insurance would bring more risk to the pool than would be commensurate with the premium they paid, whereas those who bought full coverage would be paying more than their fair share based on the risk they were contributing. The coinsurance clause provides a simple mechanism to adjust for the skewed loss distribution.

(3) The overall rate level can be lower but still adequate. This is really a corollary to the second point. By making everyone pay a premium equal to the risk for which they will be compensated, it allows for a lower average rate per thousand of coverage since it removes the possibility of anti-selection (i.e., a case where someone knowingly buys a small amount of insurance, but still expects full coverage on claims less than the policy limit).

There are also some disadvantages of this coinsurance arrangement, which include the following:

(1) The clause is not well understood by many policyholders.

(2) Because of the misunderstanding of the coinsurance clause, some costly disputes arise over its use and meaning.

(3) A policyholder who buys less than full coverage is only penalized if there is a claim, since he or she can pay a lower premium and get away with it. For example, a restaurateur may have to prove the existence of insurance coverage before being granted a license or a mortgage.

(4) The 80% coinsurance percentage (or any other percentage less than one hundred) discriminates against those who carry higher levels of insurance (which should be encouraged).

(5) With high rates of inflation in real estate (a recent problem), a homeowner may unwittingly fall below the coinsurance percentage requirement. (Note that this is why the 80% requirement is used; houses seldom rise in value by more than 25% in one year.)

(6) The use of a coinsurance percentage less than 100% may imply a recommendation to the policyholder to buy less than full coverage.

In order to make sure that the policy limit keeps up with inflation, most insurers offer an option whereby the limit is increased automatically each year, in accordance with an appropriate inflation index. If the homeowner selects this option, the coinsurance penalty is waived if the insured value falls below 80% of full coverage (and there were no unreported improvements to the property.)

Coverage B of Section I provides a specific amount of insurance on a garage and other structures on the premises, which are separate from the primary dwelling, normally equal to 10% of the dwelling coverage amount. This insurance may be increased above the standard 10% by paying an extra premium. Separate structures used for business purposes or held for rental are not covered.

The coverage on the dwelling and other buildings under the normal homeowners policy provides replacement cost coverage if the coinsurance clause has been satisfied. Otherwise coverage is for the actual cash value specified in the policy and the coinsurance provision does not apply.

Coverage C of Section I of the homeowners policy will insure the actual cash value of the policyholder's personal property and contents of the house up to a defined policy limit which is usually a percentage of the insured value of the house. The actual percentage varies among insurers but is normally either 40% or 50%. Thus, if the home is covered for $400,000, the contents would be covered up to a policy limit of $200,000 for the covered perils, if the Coverage C percentage is 50%. Many insurers also offer replacement cost coverage on contents, for an additional premium. Coverage C extends to borrowed property in the possession of the insured.

Inside limits apply to certain losses. There will be, for example, a defined limit on how much will be reimbursed in the case of loss or theft of cash (e.g., $1000). Also, there will be inside limits on the coverage for jewelry, silverware and art. If the homeowner wants full insurance on these last three items, then a ***schedule*** of the items to be insured, with their appraised values, will be attached to the policy and an extra premium will be charged for these scheduled items. In the case of loss or theft of such items, the amount paid out by the insurer will be exactly the amount set in the schedule, not the current market value of the items. This benefit is referred to as a ***valued*** benefit (since there is no loss

distribution associated with this part of the policy), such as exists in life insurance.

The coverage for the contents of the home also applies when personal items are outside of the home. If, for example, you are traveling and lose some personal assets, your homeowners policy will provide coverage for such a loss, normally anywhere in the world. Coverage on property at any other insured residence besides the main resident dwelling (e.g., a vacation home) is normally limited to 10% of the amount of Coverage C.

Coverage D of Section I provides coverage for additional living expense and loss of rental income. This coverage will pay the fair rental value for alternative accommodation while your dwelling is being repaired because of damage caused by a covered peril. It will also compensate you for loss of rental income from a part of the house which is lost while the damage is being repaired. The limit of coverage for Coverage D is normally 20% of the coverage on the dwelling.

The second major section of the homeowners policy provides liability coverage to the policyholder. Liability could arise if a third party is injured or if the property of a third party is damaged while on your property or in certain other circumstances. Before any payment is made, however, the injured party is technically required to establish negligence on the part of the homeowner, and show that there were injuries or damage that require compensation. Although this might require a full court hearing, most claims are settled out of court.

As with auto insurance, the insurer will defend the homeowner in court and pay the costs of defense. As with auto insurance, however, the insurer has the right to settle out-of-court without the insured's permission. Payments to the third party for injury or property damages are limited by the liability limits of the homeowners policy (this will be a specified limit such as $2 million), but the cost of the defense is paid over and above the payment for damages. The insurer can break off the defense of the case once it has paid costs for damages which equal the liability limits defined in the policy.

Finally, the homeowners policy will normally provide very limited medical coverage for any third party injured on your property without the need to sue to recover (i.e., on a no-fault basis).

The homeowners insurance policy is not meant to cover buildings used for commercial purposes and will so stipulate. We will not specifically discuss such commercial coverage here.

Rates for homeowners insurance vary by the home's geographic location, its construction, and its value. The geographic location parameter reflects such risk classification factors as distance to the nearest fire station, the probability of perils such as earthquake, floods, windstorms, and so on. If the probability of a defined event is very high, and would create unaffordable rates, one alternative is to exclude the named peril and offer coverage for this named peril at an extra premium. This is common practice for flood and earthquake.

Rates will also vary depending on the construction materials used in the house. Homes with untreated cedar shake shingles as roofing, for example, may have higher rates. Discounts on rates may be offered if there is a security or sprinkler system in the home, and so on.

2.4 Tenants Package

For people who rent rather than own their own homes, coverage is available in a policy called a ***tenants package***, which covers most of the items contained in a standard homeowners policy, but with modifications to reflect the lesser exposure to risk inherent in a tenant's contribution to the pool. The chance of a liability claim, for example, is far less in an apartment than on the land, or in and around the home, of a homeowner. Much of the total liability risk inherent in an apartment complex will be covered by the liability coverage carried by the owner of the apartment building. Also, if the apartment building is damaged by wind, fire or other insured peril, it is not of concern to the tenants except as to their personal possessions and some rights of tenancy. Therefore, the coverage within a tenant's package is mostly coverage for personal possessions in one's apartment or storage area. This policy will not be discussed further here.

There is also a special policy form for condominium owners that reflects their ownership interest.

2.5 Workers Compensation

Workers compensation is an example of an early introduction of no-fault insurance whereby workers gave up their rights to sue their employers in

cases of occupational accident or sickness in return for no-fault benefits on a pre-defined or scheduled basis. Workers compensation requirements are defined by legislated statutes in each state or province.

Prior to 1895, it was normally very difficult for a worker to get compensation in case of injury or illness. The worker was forced to sue the employer and prove negligence on the part of the employer. It was normally the case that the worker could not collect compensation if the worker contributed in any way to the injury or sickness (the ***doctrine of contributory negligence***), or even if the injury or sickness resulted from the negligence of a fellow worker (the ***fellow-servant doctrine***), because it was necessary to show that the employer was at fault. Further, the ability to sue was often restricted if it could be shown that the worker had advance knowledge of the inherent dangers of the job (the ***assumption-of-risk doctrine***).

Under current workers compensation laws, which exist in all states and provinces, the employer is deemed to be absolutely liable for the occupational injuries suffered by the worker, regardless of who might be at fault in the eyes of a court of law. In return, the compensation paid to the injured or sick worker is normally limited to the benefit defined in the workers compensation legislation.

Objectives of workers compensation include the following:

(1) Broad coverage of workers for occupational injury and disease.

(2) Substantial protection against loss of income.

(3) Sufficient medical care and rehabilitation services.

(4) Encouragement of safety. (Most state workers compensation plans, for example, allow experience rating whereby employers with superior claims records pay relatively lower premiums, and vice versa.)

(5) An efficient and effective delivery system for benefits and services.

In Canada, workers compensation is normally administered by a Workers Compensation Board controlled by the provincial government. In the United States, an employer can satisfy the compulsory workers compensation law by obtaining private insurance (the most common method), by self-insurance, or by use of the state workers compensation fund. Five states offer only a monopolistic state fund.

About 87% of *salaried* workers in the United States are covered by some form of workers compensation. Depending on the state, those not covered include farm labor, domestic servants, casual employment, and employees of some very small firms. Some states also exempt nonprofit educational, charitable, or religious organizations. In two states, employers can "opt-out" of workers compensation. If they do so, they may be subject to litigation by injured employees. Railroad workers in interstate commerce and seamen in the U.S. Merchant Marine are covered under the Federal Employees' Liability Act with very similar provisions and benefits.

For an injury or sickness to be covered by workers compensation, the injured worker must work in a covered occupation and have experienced an accident or disease that arose out of *and* in the course of employment. Diseases are conditions that develop over an extended period of time due to exposure to some condition related to employment. The definition of disease encompasses a variety of ailments, including hearing loss, carpal tunnel syndrome, and asbestosis. These tend to be more costly on average than traumatic (sudden) injuries, such as a fall that causes a broken leg, and are often the subject of disagreement concerning the extent to which they are work related and thus are compensable. Examples of injuries that are not covered are those arising out of driving to and from work, employee intoxication, and intentional self-inflicted injuries.

A worker can usually expect the following workers compensation benefits:

(1) Medical care benefits, which represent about 55% of the workers compensation claims by amount. Workers compensation normally provides unlimited medical care (i.e., there are normally no dollar or time limits).

(2) Disability income benefits payable to the worker after a waiting period of from 3 to 7 days. If the worker is disabled long enough, then benefits are normally paid retroactively to the date of injury. The weekly benefit is based on a percentage of the worker's average weekly wage (e.g., $66\frac{2}{3}$%) and the degree of disability. Disability Income benefits are normally non-taxable income. Most states have minimum and maximum weekly benefits, which normally adjust with the state's average industrial wage. The degree of disability is

usually classified as one of (a) temporary but total, (b) permanent and total (for which most states pay lifetime benefits), (c) temporary and partial, or (d) permanent but partial. Examples of the latter include the loss of a limb or an eye (for which a set scheduled benefit would be paid), and a back injury (a non-scheduled injury for which some benefit, which is a function of the wages lost, would be paid).

(3) Death benefits including a burial allowance plus cash-income payments to any eligible surviving dependents.

(4) Rehabilitation services and benefits. This may include vocational evaluation and training.

In the United States, workers compensation benefits are paid either by insurers or by employers who are "qualified self-insurers." Many large employers are insured, but maintain a large deductible or use retrospective rated plans, which are discussed in Chapter 5. The premium is normally a function of the payroll of the employer, the industry class of the occupation being covered (e.g., lumberjacks are charged a higher rate than are bank tellers), and so on. For small employers, all companies within a defined industry classification are charged the same rate (this is called ***class rating***). In some states, the administrative costs of the workers compensation system are paid by the state. Normally, however, in the United States virtually all costs are borne by the employers.

The benefits and eligibility requirements for workers compensation in Canada are almost the same. The primary difference between workers compensation in the United States and Canada is the common use of private insurance in the United States. In most of the rest of the world, workers compensation is almost always administered by a government agency, which also sets the rules for coverage, the rates to be paid, and the level of benefits.

2.6 FIRE INSURANCE

Homeowners and tenants package provide property and liability insurance to individual homeowners and tenants. Businesses also need property and liability insurance. Standard coverages that provide such protection are reviewed in this and the following two sections.

Fire insurance is designed to indemnify the insured for loss of, or damage to, buildings and personal property by fire, lightning, windstorm,

hail, explosion, and other perils. Coverage may be provided for both the direct loss (i.e., the actual loss represented by the destruction of the property), and the indirect loss (the loss of income and/or extra expenses due to the loss of use of the protected property). Originally only fire was an insured peril, but the number of perils insured against has gradually been expanded until it has reached the present status where even all-risks coverage can be provided, albeit with some exclusions, as previously mentioned.

The ***standard fire policy*** (SFP) is the starting point for all fire insurance coverages. The SFP covers only direct loss from fire and lightning, and at least one additional form must be attached to have a valid policy. Forms used to complete the coverage under the SFP include the following:

(1) Those that provide personal coverage (dwelling building and contents forms).

(2) Those that provide commercial coverages (general property, multiple location, and reporting forms).

(3) Those that increase the covered perils, such as the extended coverage perils of vandalism or malicious mischief, and the optional perils policy.

(4) Those that increase the covered losses, such as additional living expenses, rental value, rental income, leasehold interests, demolition expenses, consequential loss or damage, replacement costs, business interruption losses, profits and commission losses, and extra expenses.

Types of coverages written by fire insurers on separate policies, rather than by forms attached to the SFP, are called ***allied lines***. Principal allied lines include earthquake insurance, rain insurance, sprinkler leakage insurance, water damage insurance, and crop hail insurance. Risk managers of large corporations may design their own forms to meet their own specific needs.

2.7 Marine Insurance

There are two types of ***marine insurance***, ocean marine and inland marine insurance. Many ***ocean marine insurance*** policies are closely related in wording to those originally written at Lloyd's Tea House more

than 200 years ago. Similarly, when insurance forms and policies were needed for the trucking industry, modifications of marine insurance forms were used. Thus developed the name ***inland marine insurance*** for the trucking industry.

Marine insurance, like fire insurance, is designed to protect against financial loss resulting from damage to, or destruction of, owned property, except that here the covered perils are primarily those connected with transportation.

Ocean marine insurance policies provide coverage on all types of oceangoing vessels and their cargoes. Policies are also written to cover the ship-owners liability. The coverage of the basic policy applies to cargo only after it has been loaded onto the ship, but policies are frequently endorsed to provide coverage from "warehouse to warehouse," thus protecting against overland transportation hazards as well as those of the ocean.

Risks eligible for coverage under inland marine forms include the following:

(1) Domestic shipments including goods being transported by railroads, motor vehicles, or ships and barges on the inland waterways and in coastal trade. In addition, provision is made for insuring goods transported by air, mail, parcel post, express, armored car, or messenger.

(2) Instrumentalities of transportation and communication such as bridges, tunnels, piers, wharves, docks, communication equipment, and movable property.

(3) Personal property floater risks used for coverage of construction equipment, personal jewelry and furs, agricultural equipment, and animals.

2.8 Liability Insurance

In Sections 2.2 and 2.3, we pointed out that liability is an important coverage within both auto and homeowners insurance, and it provides two levels of security for the policyholder. First, in the case of injury to a third party or in the case of damage to the property of a third party, where negligence was alleged against the policyholder, the insurer will

defend the policyholder in court, if necessary, but the insurer can stop the defense when the amounts paid for damages equal the limits of the policy. Further, if the policyholder were found to be negligent and at fault, and damages were assessed to be paid to the third party for injury or property damage, then the insurer would pay these damages within the policy limits. Note that payment of damages plus the cost to defend can, and often do, exceed the stated policy limits. As stated earlier, most cases are settled out of court.

Examples of liability insurance sold as a separate coverage include ***product liability insurance***, ***errors and omissions insurance***, ***medical malpractice insurance***, ***professional liability insurance***, ***directors and officers liability***, ***employment practices liability***, ***cyberliability***, and others of a similar nature.

These coverages are characterized by low claim frequency but high value claims that often are reported many years after occurrence and take many years to settle once they are reported. In order to make these coverages more predictable by reducing the time delays, some are insured on a ***claims-made*** policy form. Under claims-made, only claims that occurred after a specific date and are reported during the policy period are covered. ***Tail coverage*** is sold for claims occurring during the period, but which are reported after the final policy period provided by the insurer.

A second characteristic of these claims is the potential for high litigation cost. In order to reduce their risk, insurers sometimes include defense within the policy limit. Therefore if there is a $2 million limit, that amount covers both defense cost and indemnity payment. If the limit is exhausted by defense costs, the insurer can withdraw.

2.9 Limits to Coverage

In all of the types of insurance outlined in this chapter there have been limits to coverage. More specifically, these limits included deductibles as well as policy limits (both overall and some inside limits). We now discuss the reasons for such limits, and some of the problems associated with them.

2.9.1 DEDUCTIBLES

Reasons for deductibles include the following:

(1) Small losses do not create a claim, thus saving the associated expenses.

(2) For larger losses, the average claim payment is reduced by the amount of the deductible which is translated into premium savings.

(3) The fact that the deductible puts the policyholder at risk provides an economic incentive for the policyholder to prevent a claim.

(4) The policyholder can optimize the use of limited premium dollars by using the deductible to save money where the value of the coverage is not as great (i.e., in terms of its utility).

Problems associated with deductibles include the following:

(1) The insured may be disappointed that losses are not paid in full. Certainly, deductibles increase the risk for which the insured remains responsible.

(2) Deductibles can lead to misunderstandings and bad public relations.

(3) Deductibles may make the marketing of the coverage more difficult.

(4) The insured may inflate the claim to recover the deductible, which is unfair to the honest policyholders who will pay the resulting higher premium.

There are several types of deductibles, including the following:

(1) ***Fixed dollar deductibles***, which apply to each claim.

(2) ***Fixed percentage deductibles***, which may be a percentage of either the loss or the policy limit, that apply to each claim. A fixed percentage deductible is usually combined with a minimum dollar deductible so the insurer does not need to handle small claims.

(3) A ***disappearing deductible***, as explained in Section 2.3 on homeowners insurance. If the loss is less than a dollars, the insurer pays nothing. If the loss exceeds b dollars, then the insurer pays the loss in full. If the loss is between a and b, then the deductible is reduced pro rata or linearly between a and b (see Example 2.1). The complexity of the disappearing deductible and the difficulty in making it understandable has resulted in its decreasing use.

(4) A ***franchise deductible***, whereby if the loss is less than n dollars the insurer pays nothing, but if the loss equals or exceeds n the claim is paid in full. This is just a ***cliff disappearing deductible***. This type of deductible used to be common in ocean marine insurance, but is seldom used today.

(5) Health insurance policies or medical expense insurance policies (not discussed in this book) often use a ***fixed dollar deductible per calendar year*** (as opposed to a per loss deductible). Often the policyholder is able to choose among a variety of deductibles with the premium going down as the deductible goes up. This deductible can sometimes be a dollars for an individual policyholder, but b dollars for a family under family coverage. This type of deductible is not widely used outside of health insurance.

(6) Disability income and sickness insurance benefits often have an ***elimination period***, which is the period from the time of the disablement to the date that disability benefits begin. This is common in workers compensation. Sometimes the elimination period differs depending upon whether the cause of the disability is an accident or a sickness. If so, it is common to have a shorter elimination period for accidents than for sicknesses. As noted in Section 2.5 on workers compensation, if the disability continues beyond a defined period of time, benefits will then be paid retroactively to the first day of disability. If the retroactive qualification period equals the elimination period, this arrangement is equivalent to a franchise deductible.

2.9.2 Policy Limits

An insurer can have a variety of reasons for placing a limit on the coverage provided in a policy, including the following:

(1) The limit clarifies the insurer's obligation. (Note that workers compensation medical coverage is unlimited.)

(2) In the context of risk, setting a policy limit provides an upper bound to the loss distribution for the insurer and lessens the risk assumed by the insurer. This, in turn, decreases the probability of insurer insolvency. Having policy limits also decreases the premium that must be charged for the basic coverage.

(3) Having policy limits allows the policyholder to choose appropriate coverage at an appropriate price (the premium will be lower for lower policy limits).

As has been discussed throughout this chapter, a policy can have more than one limit, and, overall, there is more than one way to provide for policy limits. In a homeowners policy, for example, there will be a defined limit for the liability coverage. The dwelling coverage will be limited by the value of the dwelling determined by the policyholder and the agent. The contents coverage limit is a percentage of the dwelling limit. There will be scheduled limits on the coverage provided for jewelry, silverware, and so on. There can also be inside limits for other coverages, such as a limit on the insurance for stolen cash.

Remember that the total of all loss adjustment expenses (e.g., legal costs) and the payment of damages can exceed the policy limit. Only the payment of damages is subject to the policy limits.

2.10 CONCLUSION

This chapter has provided a generic description of several of the most important insurance coverages provided by property/casualty insurers. The descriptions were very general, and should apply to these coverages worldwide. The coverages described are only a sample of coverages available. Other coverages include ***boiler and machinery insurance, contract surety insurance, business interruption insurance***, and many others. Each coverage has its own particular policy legal requirements and limits, and, as a result, each has its own unique ratemaking methodology.

Readers of this textbook are encouraged to review their own auto and homeowners (or tenants package) policies carefully to understand the legal restrictions that exist within their own coverages.

2.11 EXERCISES

Section 2.2

2.1 Differentiate between the specified perils and the all-risks or comprehensive approaches to coverage. Is the all-risks approach truly all-risks?

2.2 Define clearly the concepts of (a) salvage and (b) subrogation. Describe the effects that these concepts have on insurance premiums.

Section 2.3

2.3 What is the significance of the doctrine of proximate cause in determining covered losses?

2.4 (a) List and explain the objectives of the coinsurance clause.
(b) List the disadvantages of the coinsurance clause.

2.5 Find the amount of the loss, given the following information:

Amount of insurance purchased	400,000
Coinsurance requirement	80%
Property's insurable value at time of loss	800,000
Property's insurable value when policy was purchased	700,000
Amount of loss paid by the insurer	320,000

2.6 A building is worth 200,000 just before a loss. It is insured by a policy that has an X % coinsurance clause. The amount of loss is 10,000 and the insured carried 120,000 of coverage. The insurer pays 7,500. Calculate X.

2.7 Assume that in Exercise 2.6 the coinsurance requirement was 70% and that there was a loss of 175,000. How much does the insurer pay?

2.8 Polly C. Holder has coverage with a linearly disappearing deductible. Up to 250 of claim, the insurer pays nothing. Beyond 1000, the insurer pays all. What does the insurer pay on a claim of 750?

Section 2.5

2.9 With respect to workers compensation insurance, define and differentiate the following doctrines:
   (a) Contributory negligence
   (b) Fellow-servant
   (c) Assumption-of-risk

2.10 List the objectives of workers compensation.

2.11 Outline briefly the normal benefits provided under a workers compensation plan.

Section 2.9

2.12 List the advantages and disadvantages of deductibles.

2.13 In each of the following cases, what will the insurer pay on a claim of 12,000?

   (a) A 20% deductible and a policy limit of 12,500.

   (b) A straight deductible of 1000 and a policy limit of 10,000.

   (c) A linearly disappearing deductible such that a claim of 5000 has no loss payment, but a claim of 15,000 is paid in full.

2.14 A decision maker is faced with a random loss that has a uniform distribution over the interval $0 < X < 10$. If she wishes to pay a premium of 2, then the optimal coverage requires a deductible of d. Assuming no expenses, calculate d.

2.15 A company provides a coverage whose loss distribution is uniform over the interval $0 < X < 5000$. If the company moves from a deductible of 250 to a deductible of 500, how much will the expected loss payments be reduced?

2.16 List and explain the reasons for the use of policy limits.

LOSS RESERVING[1] ○ 3

3.1 INTRODUCTION

One of the two most important functions the actuary performs in the property/casualty insurance industry is ***loss reserving***. The other function is ratemaking and will be discussed in Chapter Four.

Perhaps the one of greater importance, loss reserving is the function whereby the actuary determines the present liability associated with future claim payments. In fact, this function is so important that most jurisdictions now require by law that a qualified actuary attest to the adequacy and appropriateness of the insurer's loss reserves and, in most jurisdictions, that there is sufficient assets/surplus to cover those liabilities. In the United States, qualified actuaries are only required to opine on the "reasonableness" of the recorded reserves. These reserves are by far the largest liabilities carried by property/casualty insurers. Since these balance sheet reserves are based on claims settlements that will happen in the future for claims that have occurred as of the statement date, there is a great deal of uncertainty connected with their determination.

A loss reserving actuary must act as a ***fiduciary*** for at least two parties. First, and of primary importance to the regulators, the actuary is protecting the rights of the insurer's policyholders. By attesting that the insurer has set aside enough money to pay all future benefits on obligations that already exist, the actuary is providing policyholders with assurance that their benefits will be paid. If the reserves are appropriate, there should be enough funds to guarantee the existing obligations to the policyholders even if the insurer were declared insolvent.

Second, the actuary provides an important service to the insurer, its owners, and any potential future owners. By attesting to the insurer's

[1] Many tables show three-decimal accuracy. The actual calculations shown here were done on EXCEL™ spreadsheets with much higher accuracy.

solvency, the actuary is confirming that reported profits are real, and they can be distributed or reinvested. The actuary's certification of liabilities also provides evidence to potential shareholders, or buyers interested in acquiring the insurer, of the level of solvency and the adequacy of the stated liabilities. (Note that a true measure of solvency requires a proper evaluation of the insurer's assets as well.)

The reserving actuary also provides data to the insurer that permits the accurate calculation of incurred claims (paid claims plus reserves), which become primary building blocks for the ratemaking actuary in setting rates for future exposure periods. Ratemaking uses this historical experience to project expected claims for future periods. Unless the claims data for the historical experience period are complete and developed to their ultimate estimated values, proper ratemaking is impossible.

In summary, accurate actuarial loss reserving is essential for a variety of reasons and an actuarial opinion by a qualified actuary is required by law in most jurisdictions.

3.2 How Outstanding Claim Payments Arise

Figure 3.1 shows the history of the incurred loss development for one claim. Figure 3.2 portrays events related to this hypothetical claim. The abbreviation "CY" is used to represent calendar year.

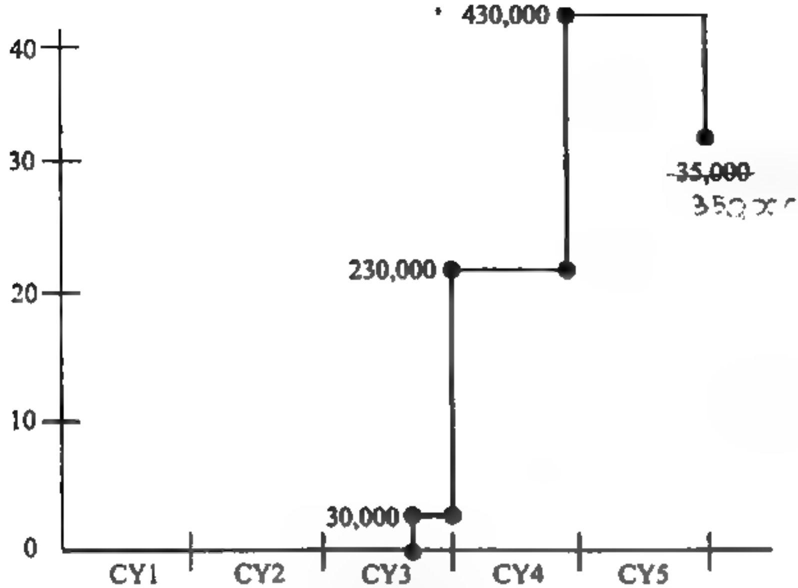


Figure 3.1

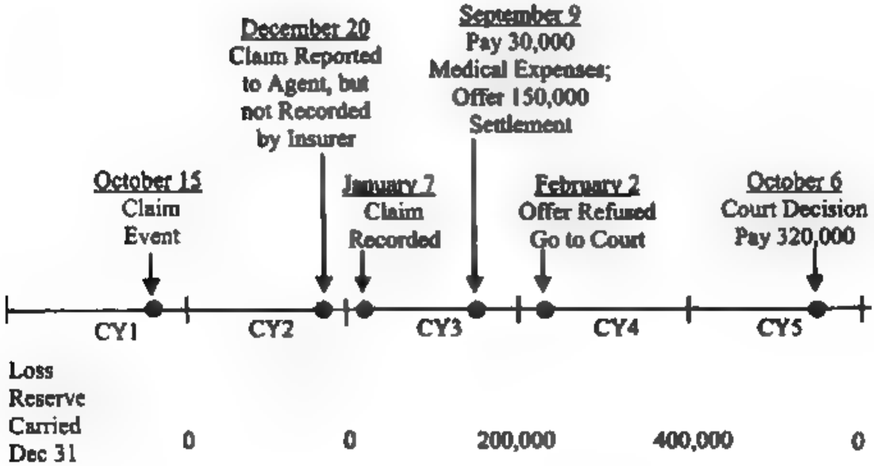


Figure 3.2

In the example of Figure 3.2, although the occurrence date was October 15, CY1, the insurer was first informed of the claim on January 7, CY3. This can easily happen, as, for example, with auto third-party bodily injury, where the effects of a back injury are not clear for several months. Another common example would be with a medical malpractice suit. What is unrealistic in this example is that a court case could be scheduled, held, and concluded in such a short period since many suits are filed just before the statute of limitation would apply.

Depending on the size of the claim and the legal complications surrounding it, a claim file could remain open for ten to twenty years. During that time there could be many changes in the estimate of the expected liability.

In this regard, the size of, and difficulty in estimating, outstanding liabilities varies widely by coverage. Auto collision and homeowners property claims are normally settled in a matter of weeks. Generally, after two or three years from occurrence the total amount of the claim is known with a high degree of confidence.

Claims for auto bodily injury and medical malpractice, on the other hand, can easily take ten to twenty years to be settled, and unfortunately, the largest claims settle last. This leads to a great deal of uncertainty in the estimation of the total amount of a claim, and therefore uncertainty in the estimation of the loss reserve liability.

There are at least two sources of uncertainty regarding unpaid claims. First, the estimated claim payments to be made on known claims can and do change from time to time until finally settled. Second, there may be claims, especially at early durations that have been incurred but not yet reported to the insurer, as is the case on December 31, CY1 and December 31, CY2 for the claim illustrated in Figure 3.2.

Just as in the ratemaking methodology outlined in Chapter Four, the normal method used for loss reserving is to review data from an historical experience period, and then use the patterns presented by that data to estimate expected future events.

3.3 Definition Of Terms

3.3.1 Basic Elements

(1) A ***claim frequency distribution***, which has been developed from recent experience data. The ***average claim frequency*** is defined as

$$f = \frac{\text{number of incurred claims}}{\text{units of earned exposure}}. \tag{3.1a}$$

Note that it is possible for a loss to occur without the policyholder filing a claim (e.g., a loss smaller than the policy deductible).

(2) A ***loss distribution***,[2] which models the severity of losses that are incurred. The ***average loss severity*** is defined as

$$\begin{aligned} S &= \frac{\text{dollars of incurred losses}}{\text{number of incurred claims}} \\ &= \text{average payment per insurred claim} \end{aligned} \tag{3.1b}$$

Total incurred losses equals losses paid-to-date plus the unpaid loss reserve being carried (for accident and policy year data), as will be explained later.

(3) An ***aggregate loss distribution***; that is, the combination of the frequency and severity distributions or developed directly from total losses.

(4) A rate of interest, i, or force of interest, δ.

(5) Payout pattern - the times at which the payments are made. This would be derived from an aggregate loss distribution of claim payments.

(6) The loss cost is the expected value of the incurred losses per unit of exposure. It is also called the ***pure premium*** by property/casualty actuaries and the ***net premium*** by life actuaries.

We further define the ***loss cost*** per unit exposure as

[2] For a full discussion of loss distributions, see *Loss Models: From Data to Decisions*, by S.A. Klugman, H.H Panjer, and G.E. Willmot, John Wiley & Sons, New York, 2012.

$$\begin{aligned}\text{Loss Cost} &= \text{Average Claim Frequency} \times \text{Average Loss Severity}\\ &= \frac{\text{number of claims}}{\text{units of earned exposure}} \times \frac{\text{dollars of incurred losses}}{\text{number of incurred claims}}\\ &= \frac{\text{dollars of incurred losses}}{\text{units of earned exposure}}.\end{aligned} \tag{3.2}$$

(7) For economic modeling of cash flows related to loss reserves, the input variables would be: total reserves to be paid out (frequency times severity); an interest rate assumption and a payment pattern.

Historically, in most life insurance and pension reserve valuations, variation from expected values generally occurs only in the survival model and the rate of interest, since the size of claim is either a constant defined in the policy or a well-defined function (e.g., a percentage of salary). Hence, for the life actuary, a loss distribution may be of little concern.

In property/casualty insurance, sources of variation from the expected lie in the claim frequency distribution (experienced claim frequencies are used to model the underlying unknown claim frequency distribution) and the loss distribution (experienced loss severity values are used to model the underlying unknown loss distribution). For lines of business where claims are settled relatively quickly (e.g., health insurance, auto collision, or homeowners), possible variation in the rate of interest is of little consequence. In lines such as liability insurance, however, the length of time needed to settle the larger claims can easily exceed ten years, and more attention is paid to the interest component in jurisdictions where discounting of loss reserves is allowed.

3.3.2 Individual Claim File Estimates

As soon as the field adjuster is aware of a pending claim, a ***claim file*** will be established. That file will contain information pertinent to the claim, such as date of accident, date of claim, assigned lawyer, examining physician, payments-to-date, and so on. The field adjuster is expected to estimate as early as possible, and to regularly update the expected ultimate claim payment. In this regard, the adjuster will take into account the following information:

(1) The severity of the loss associated with the claim.

(2) The likely time to settlement and to final payment.

(3) Inflation between the accounting date and the expected time of final payment.

(4) Recent changes in claim settlement or payment patterns, including any changes in court adjudications or legislation.

(5) Other pertinent information as it becomes available.

The aggregate of the individual claim file estimates (split by line of business and accident year) is called the ***case reserves.***

3.3.3 Gross IBNR

Case reserves are only part of the total reserve requirement. In addition to case reserves, the following items are part of the total reserves:

(1) Provision for future development in known claims (i.e., adjustments of case reserves).

(2) Provision for claim files that are closed but may reopen (e.g., workers compensation).

(3) Provision for claims incurred but not reported (pure IBNR).

(4) Provision for claims reported but not recorded (RBNR).

The total of these four components of the total reserve is normally called the ***bulk reserve*** or the gross ***IBNR reserve.***

The reserves in the claim example in Figure 3.2 show the individual claim reserve for this claim—the case reserves. The Gross IBNR is not developed on an individual claim basis; so cannot be shown on Figure 3.2. For property casualty insurers, gross IBNR is developed in bulk typically by larger homogeneous reserving groups (i.e., malpractice occurrence policies) to capture the elements above as they would have emerged historically across all the claims of a similar nature.

3.3.4 Paid Loss Development

The change in cumulative payments made to date on a defined set of claim files (split by line of business and accident year) between successive valuation dates is called the ***paid age-to-age loss development***. The change from one specified evaluation date (or age) to the ultimate payment amount is called ***age-to-ultimate loss development.*** Normally we express loss development through loss development factors. The age to age paid loss development factor at duration j would be the ratio of the cumulative

payments at duration j to the cumulative payments at duration $j-1$. It is possible for paid loss development factors to be less than one because of salvage and subrogation.

3.3.5 Incurred Loss Development

The change in cumulative incurred claims (paid-to-date plus outstanding reserve estimates) on a defined set of claim files on successive valuation dates is called the ***incurred age-to-age loss development***. Corresponding to paid loss development factors, we will also have incurred loss development factors. It is possible for incurred loss development factors to be less than one if the claim file estimates are conservative or because of salvage or subrogation. Incurred loss development factors greater than one indicate that claim file estimates were inadequate, due to gross IBNR.

3.3.6 Salvage And Subrogation

As explained in Chapter Two, if a policyholder has a car accident and the car is "written off," the policyholder will receive a check for the value of the car at the time of claim. From that point on, the car is owned by the insurer. The right to any salvage value belongs to the insurer, and will appear in the accounts as a recovery of costs.

Similarly, if a policyholder has a house fire, the insurer pays for the house to be repaired or rebuilt within the limits of the policy. If it can be shown that the fire was caused by the negligence of a third party (e.g., faulty wiring), the insurer has the right under subrogation to pursue recovery in a law suit against the third party. Again, because of this subrogation recovery, the final development in many claim files is negative, leading to a final loss development factor less than one.

3.3.7 Loss Adjustment Expenses

Reserves for the expenses associated with the loss adjustment process must also be provided. Such ***loss adjustment expenses*** are classified as ***allocated*** (ALAE) or ***unallocated*** (ULAE).

Allocated loss adjustment expenses, such as lawyers' fees, can be associated with a specific claim. These expenses become part of the total claim cost, and the reserving techniques described here can automatically include a reserve for allocated loss adjustment expenses.

Unallocated loss adjustment expenses include general salaries, heat, light, rent, and so on, needed to support the claims process, which cannot be associated with a particular claim. Reserves for these expenses are allocated to calculated reserves at year end using mathematical formulae. These allocation techniques are beyond the scope of this text.

The classification between allocated and unallocated loss adjustment expense can vary between insurers. The important point is that the definition should remain consistent over time.

For statutory financial reporting purposes in the United States, a new classification of loss adjustment expenses was introduced effective 1/1/1998 as Defense and Cost Containment (DCC) and Adjusting and Other (AO) expenses which moved some ALAE to ULAE and vice versa. Again more important than the definition is the importance of reviewing historical experience on a consistent basis. Many U.S. insurers and reinsurers still continue to use the prior definitions in practice.

3.3.8 Fast Track Reserves

For high frequency, low severity, fast-closing lines of business such as auto collision, claim adjusters often use a ***fast track average reserve*** based on recent past experience adjusted for trends, provided by the home office actuary, each time a claim file is opened. If a claim file remains open beyond a certain period, an individual estimate will replace the fast track average.

The use of fast track reserves saves time and effort, compared to the use of individual estimates, for claims that are small and close quickly. The reserves will be as accurate as the average value that has been provided by the actuary.

3.4 Professional Considerations

Setting loss reserves is not the job of a technician, but of a professional actuary. We cannot enter data into a computer software package, press a button, and accept the reserve estimate that results. A considerable degree of judgment is required.

The actuary, whether the insurer's employee or an independent consultant, must become intimately involved with the insurer for which reserves are being established. He or she must have full knowledge of changes in mix of business, product pricing, claims administration procedures, and overall management goals and attitudes. Has there been a change in the company's

philosophy in settling claims (e.g., a more aggressive use of court defense)? Has there been a change in the company's retention limit in its reinsurance contracts?

Beyond a knowledge of the internal workings of the insurer, the actuary must be fully cognizant of external factors such as the rate of inflation, changes in court adjudications, changes in legislation, and so on. No formula or software approach can replace the human involvement and professional judgment that the actuary brings to the loss reserving process.

The format of the data needed for loss reserving varies widely among insurers. Generally, however, files should contain the following information:

(1) Line of business (e.g., homeowners, fire, automobile liability).

(2) Date of loss or occurrence.

(3) Report date (i.e., date reported to insurer).

(4) Policy issue date.

(5) Policy number.

(6) Claim number.

(7) All payment and change-in-reserve data with dates.

(8) Other data that will vary widely among claims.

The actuary must know all the definitions used by the insurer. As an example, if a single-car accident results in two third-party bodily injury claims and one collision claim, some insurers will assign three claim files and a claim count of three. Others will assign two claim files and a claim count of two, and, rarely, an insurer might count this as one claim. With this level of variability in data, the actuary must work closely with the insurer's personnel before starting any quantitative analysis.

The actuary is responsible for establishing data collection techniques and categories that will satisfy the loss reserve analysis. The data should be separated into homogeneous categories (e.g., blocks of business with similar frequency and severity patterns). Sometimes in order to obtain greater homogeneity, claims within a policy may be split into more than one segment. Homeowners claims, for example, should be split between property and liability. The property claims can be further split into each of the four coverages (A, B, C, D) of the homeowners policy described in Section 2.3. Worker compensation claims are often split into medical and

indemnity (wage loss) components. Once again, the actuary must compromise between data homogeneity and the necessity of having enough data in any cell to achieve statistical credibility. (The reader's understanding of the type of data normally required will become clearer as the chapter proceeds.)

It is the responsibility of the actuary to review the accuracy of the data. Normally the actuary will then develop reserve estimates using more than one method. A critical part of loss reserving is the evaluation of the inevitable conflicting results among the various methods, and the ability to reconcile or explain the differences. Through this process the actuary attempts to achieve a loss reserve estimate that is as accurate as possible.

3.5 Checking the Data

The loss reserving techniques outlined in this chapter are all based on the assumption that past loss payment patterns can be used to model the future. In this regard, the actuary should review the data to see that the patterns of loss payments are consistent over the years and do not contain any anomalies. By analyzing any inconsistencies, the actuary will discover important information about the data and the insurer that will aid in establishing more accurate reserves. The actuary will work with others responsible for the data systems in the company, the claim department and sometimes with the insurer's auditor in the verification of the data.

In studying the reserving techniques outlined in this chapter, the reader should keep in mind that the first action of the actuary receiving any data analyses is to check for pattern consistency and aggressively pursue a reconciliation, or explanation, of any inconsistencies in multiple-source data. In this regard, it is worthwhile noting that the impact of any calendar year change will present itself across data diagonals, as will be seen later.

Finally, it is essential that the reserving actuary carefully document the findings of these reviews and the reasons for any subjective changes made in the statistical calculations. This documentation should become part of the actuary's official report to management and regulators.

3.6 Loss Reserving Methods

There are many ***loss reserving methods***. This section will review families of methods that are currently popular in practice.

3.6.1 Case Reserves Plus

The first estimate of the loss reserve liability is the total of all existing individual claim file estimates, which we have called the case reserves. In addition, the gross IBNR reserve, the sum of the following four items, must be estimated.

(1) Pure IBNR

(2) RBNR (Reported But Not Recorded)

(3) Future adjustments of case reserves on known claims

(4) Files that are closed but may reopen.

Historically, the use of this approach was not applied consistently and in some instances led to some manipulation of company results. For insurers not using actuaries, the loss reserve liability was often set arbitrarily by adding to the case reserves some percentage that approximated the gross IBNR requirement based on past experience. Because of the subjective nature of this ***case reserves plus*** approach, some insurers would set aside a larger gross IBNR reserve in profitable years (thereby decreasing their net profit and their taxes). Then they would set aside a smaller gross IBNR reserve in unprofitable years to retain a profitable bottom line. Unfortunately, without the discipline of sound actuarial methodology, the gross IBNR for some insurers became the year-end balancing item. This led to underestimates of required reserves for insurers that were consistently unprofitable. In the long run this led to the insolvency of several insurers, and policyholders were not paid their promised benefits.

Firms using actuaries to set loss reserves today would not use such tactics. Most jurisdictions now require that a qualified actuary certify the adequacy or reasonableness of the annual statement loss reserve liability which serves to mitigate imprudent tactics.

The case reserve plus method is a sound methodology when assumptions are documented and applied consistently. This method would be appropriate for some unique lines of business, but generally would be used as one method among others the actuaries may employ.

3.6.2 The Expected Loss Ratio Method

A second method of loss reserving is the ***expected loss ratio method***. The individual responsible for setting the loss reserve for a particular line of business and policy period will calculate (or estimate) the expected ultimate loss ratio for that block of business. That expected loss ratio, multiplied by the appropriate earned premium figure, will produce the estimated ultimate

losses. The loss reserve for that block of business will then be the estimated ultimate losses minus the losses paid-to-date.

Actuaries most often rely on earned premiums when estimating ultimate losses for a policy period for a particular valuation date, such as 9/30. Written premiums refer to the actual amount of premiums written in the policy period, whereas earned premiums reflect the portion of the written premiums that were earned for the policy period as of the valuation date.

Reviewing the steps in the calculation, we first determine E[(Loss Ratio)$_{i,j}$], where i denotes line of business and j denotes policy period. We then find

$$\begin{aligned}&(\textit{Estimated Ultimate Losses})_{i,j}\\ &\quad = (\textit{Expected Loss Ratio})_{i,j} \times (\textit{Earned Premium})_{i,j},\end{aligned} \tag{3.3}$$

which leads to

$$\begin{aligned}&(\textit{Estimated Loss Reserve})_{i,j}\\ &\quad = (\textit{Expected Ultimate Losses})_{i,j} - (\textit{Losses Paid-to-Date})_{i,j},\end{aligned} \tag{3.4}$$

from which is found

$$\textit{Total Estimated Loss Reserve} = \sum_{i,j} (\textit{Estimated Loss Reserve})_{i,j}. \tag{3.5}$$

There are several considerations associated with this method. It should not be used as the loss reserving method for an entire company portfolio, because if the expected loss ratio is manipulated by management, inappropriate or inadequate reserves can result. Furthermore, the reserves produced by this method can be illogical if the ratio is applied without careful thought. Suppose, for example, an insurer has experienced a 70% loss ratio for a particular line of business for many years, and in the past year it took a significant rate reduction for this line of business. If we simply apply the 70% historical loss ratio to the new smaller earned premium figure, the result will be a sharply decreased loss reserve liability. If, however, the rate decrease were the result of market forces, rather than an expectation of decreased future losses, then a lower loss reserve makes no sense. Also what if you have experienced a large loss in a policy period that exceeds that 70% expected loss? The expected loss method used in its purest form would not reflect this in its expected loss selection. For these reasons, the expected loss ratio method should be used with great care.

There are instances, however, when it may be the only possible method, such as for a new line of business. In the early years, before there are any claims data to analyze, the expected loss ratio method may be the only legitimate approach to estimating the loss reserve liability.

For certain lines of business, government regulations may define minimum loss ratio targets that the loss reserves must satisfy. That is, the loss reserves cannot result in loss ratios for those lines that are less than the government-mandated minimums.

The expected loss ratio method can also be used as a reasonableness check for more sophisticated methods, such as the Bornhuetter Ferguson method discussed in Section 3.6.4.

3.6.3 The Chain-Ladder Or Loss Development Triangle Method

Most actuaries today include in their analysis some form of the ***chain-ladder method***, also called the ***loss development triangle method***. This method is best explained through an example.

Table 3.1, below, lists the payments actually made for a certain line of business at progressive development durations. The entry for accident year *Z* and development year 0 shows all dollars paid in calendar year *Z* on claims with accident date in year *Z*. Similarly, the entry for accident year *Z* and development year 1 shows the dollars paid in calendar year *Z*+1 on claims with accident date in year *Z*.

Table 3.1

Incremental Loss Payments (000) by Development Year								
	Development Year							
Accident Year (AY)	0	1	2	3	4	5	6	7
AY1	5,445	3,157	2,450	1,412	600	352	431	185
AY2	5,847	3,486	1,366	848	1,045	1,054	369	
AY3	5,981	4,854	1,948	2,554	1,680	489		
AY4	7,835	4,453	3,888	3,335	2,088			
AY5	9,763	6,517	3,563	3,984				
AY6	10,745	6,184	4,549					
AY7	14,137	8,116						
AY8	15,162							

Table 3.1 can be transformed into Table 3.2 by aggregating the incremental payments.

Table 3.2

Cumulative Loss Payments (000) through Development Years								
	Development Year							
Accident Year	0	1	2	3	4	5	6	7
AY1	5,445	8,602	11,052	12,464	13,064	13,416	13,847	14,032
AY2	5,847	9,333	10,699	11,547	12,592	13,646	14,015	
AY3	5,981	10,835	12,783	15,337	17,017	17,506		
AY4	7,835	12,288	16,176	19,511	21,599			
AY5	9,763	16,280	19,843	23,827				
AY6	10,745	16,929	21,478					
AY7	14,137	22,253						
AY8	15,162							

These cumulative payments can now be analyzed for development patterns. From the cumulative payments, we calculate the age-to-age loss development factors (sometimes called ***link ratios***) presented in Table 3.3, where each entry is the ratio of successive development year cumulative payments. These calculations are explained in the section that follows.

Table 3.3

Age-to-Age Paid Loss Development Factors Based on Cumulative Payments							
	Ratio of Successive Development Years						
Accident Year	1/0	2/1	3/2	4/3	5/4	6/5	7/6
AY1	1.580	1.285	1.128	1.048	1.027	1.032	1.013
AY2	1.596	1.146	1.079	1.090	1.084	1.027	
AY3	1.812	1.180	1.200	1.110	1.029		
AY4	1.568	1.316	1.206	1.107			
AY5	1.668	1.219	1.201				
AY6	1.576	1.269					
AY7	1.574						
Average	1.625	1.236	1.163	1.089	1.047	1.030	1.013
5-Year Average	1.639	1.226	1.163	1.089	1.047	1.030	1.013
Mean	1.615	1.239	1.172	1.092	1.044	1.030	1.013

Several observations about the above tables are required. The triangles presented are based on ***paid loss data***, but it is possible to present

incurred loss data in an identical format. Hence, the actuary will usually create both paid-loss- and incurred-loss-development triangles. The paid loss data are purely objective, representing actual payments with no subjective reserve estimates. Incurred loss data include subjective reserve estimates, but the actuary would not want to ignore the extra information available in such data.

Generally it is a good idea to do both a paid-loss-development and an incurred-loss-development analysis. In theory, the values of expected ultimate payments and expected ultimate incurrals should be the same, since once all claims are completely paid, and there are no reserves, the incurrals have all become paid. In practice, the reserve values derived from the expected ultimate values of these two bases will differ, and the reconciliation of the difference will assist in defining the loss reserve estimate.

An increase in retention limits, for example, or changes in legislation or court adjudications will be reflected immediately in an incurred-loss-development triangle, but it may take many years before they are reflected in a paid-loss-development triangle.

The mathematics involved in the chain-ladder technique is identical for paid and incurred data, but the resulting numbers will look quite different. Depending on the level of conservatism in the claim file estimates, many of the age-to-age and age-to-ultimate development factors in an incurred-loss-development triangle beyond a certain age or duration will be close to 1.000.

Loss development triangles such as those in Table 3.3 demand a great deal of time and analysis. Remember, the actuary will first review the data for any inconsistencies. The actuary will then review the patterns in the columns of loss development factors and the data along each diagonal, since the diagonals represent calendar years. If, for example, the insurer decides to speed up its claim payments in a particular calendar year, the result would be reflected in consistently elevated paid loss development factors along one diagonal. But speeding up claims payments should lead to decreased loss reserves, whereas increased loss development factors will lead to increased loss reserves. Having investigated the source of this anomaly, the actuary may decide to remove or adjust that particular diagonal in the analysis. Other examples will be cited later in this chapter to show that such triangles require careful professional analysis.

In all of the examples that follow, we will assume that all loss development factors beyond those given for the oldest accident year are equal to 1.000. That is, for the oldest accident year we assume no loss development beyond the last development year printed in the example. For many lines of business, the actuary must spend much time considering the value of the loss development factors that should be used to model the tail of the loss payment pattern for which the actuary may not have data. This often requires using data compiled by a statistical agency. As an example, it is noteworthy that the first workers compensation claim on the New York State Insurance Fund, which occurred in 1914, was still open in 1990.

Once the data have been reviewed for consistency and reconciled to past analyses and company systems, and modified as appropriate until the actuary concludes the data is acceptable, the actuary can proceed to the actual calculation of the loss reserve.

The chain-ladder method develops the loss reserve in a three-step process. First, single age-to-age column factors are chosen to model the loss development indicated by existing experience data. The selected patterns of loss development are then projected to create the lower half of the loss development triangle, so the model can be used to estimate the expected ultimate payments for each accident year. Finally, the expected ultimate payments less payments-to-date represent the reserve requirement. The basic assumption of the chain-ladder method is that the past is sufficiently indicative of the future, and that there exists a consistency in items such as settlement patterns, reserve adequacy, and business written.

Many methods exist to calculate a single age-to-age loss development factor for each column of Table 3.3, three of which are denoted: average, five-year average, and mean.

The ***arithmetic average*** is the average of all loss development factors in the column being analyzed. The ***five-year arithmetic average*** uses only the five latest entries for columns 1/0 and 2/1, the logic being that only relatively recent indications should be used in setting loss development patterns. The ***mean***, or ***volume-weighted average***, for column 1/0 is the sum of the seven entries from development year 1 in Table 3.2 divided by the sum of the seven corresponding entries in development year 0. Similarly, the mean for column 2/1 is the sum of the six entries from development year 2 in Table 3.2 divided by the sum of the six corresponding entries in development year 1, and so on. (It would also be possible to calculate a 5-year mean). The

mean is just a volume-weighted average, where the weights are the amounts in Table 3.2. This has some intuitive appeal, in that years with more losses should have greater credibility and hence greater weight. Further, because of normal growth and inflation, this method would normally give greater weight to more recent data.

Other actuaries might choose to weight the last five data points with weights of 1, 2, 3, 4, 5 (or other weights), which give more weight to more recent data. Some actuaries fit linear models to the existing column data, and derive the missing lower half of the rectangle from the linear trend of the upper half, after adjusting for frequency and exposure changes.

Tables 3.4, 3.5, and 3.6, presented on the following three pages, show the calculation of the total reserve using the arithmetic average, the 5-year arithmetic average, and the mean modeling assumption, respectively. In these calculations, it is assumed that for each accident year all payments are made by the end of development year 7. Although in the examples each column was modeled by a single loss development factor produced by the respective analysis, different factors could be used for every missing entry in the lower half of the rectangle. This is common when there is a significant change in the business written over the years. Whatever method is used should be well documented and fully disclosed in the accompanying actuarial report.

In terms of statistical modeling techniques, the chain-ladder method is open to criticism. Assume, for example, that we can obtain complete loss development data for n accident years. This will produce a loss development triangle with $n-1$ columns. Assume also that the method chosen to derive the lower half of the loss development triangle uses one factor to model each column, as illustrated above. These factors will then be applied to the last n payment points, one for each accident year (i.e., the diagonal of Table 3.2), to estimate the expected ultimate payment per accident year. In terms of statistical modeling, the chain-ladder method provides the actuary with a model using $2n-1$ parameters. ($n-1$ loss development factors applied to the last n payment points)

Stability is not a characteristic of such a model. If we have a change in reserving philosophy at the management level, or one or two large claims in a particular year, the actuarial reserve estimate can change significantly. If, for example, we were to increase by 10% the cumulative payments for accident year AY7, development year 1, from 22,253 to 24,478, the total reserve using the 5-year average modeling assumption would rise 4.1%. (Compare Table 3.7 to Table 3.5; the total loss reserve increases from 58,180 to 60,552.)

Table 3.4

Estimated Paid Losses and Loss Reserves by Accident Year,
Based on Average Paid Loss Development Factors Derived in Table 3.3

Accident Year	Development Year 1	2	3	4	5	6	7	Estimated Ultimate Losses	Paid-To-Date	Estimated Loss Reserve
AY1								14,032	14,032	0
AY2							14,197	14,197	14,015	182
AY3						18,031	18,266	18,266	17,506	760
AY4					22,614	23,293	23,595	23,595	21,599	1,996
AY5				25,948	27,167	27,982	28,346	28,346	23,827	4,519
AY6			24,979	27,202	28,481	29,335	29,716	29,716	21,478	8,238
AY7		27,505	31,988	34,835	36,472	37,566	38,055	38,055	22,253	15,802
AY8	24,638	30,453	35,417	38,569	40,382	41,593	42,134	42,134	15,162	26,972
TOTAL								208,341	149,872	58,469

Table 3.5

Estimated Paid Losses and Loss Reserves by Accident Year,
Based on 5-Year Average Paid Loss Development Factors Derived in Table 3.3

Accident Year	Development Year 1	2	3	4	5	6	7	Estimated Ultimate Losses	Paid-To-Date	Estimated Loss Reserve
AY1								14,032	14,032	0
AY2							14,197	14,197	14,015	182
AY3						18,031	18,266	18,266	17,506	760
AY4					22,614	23,293	23,595	23,595	21,599	1,996
AY5				25,948	27,167	27,982	28,346	28,346	23,827	4,519
AY6			24,979	27,202	28,481	29,335	29,716	29,716	21,478	8,238
AY7		27,282	31,729	34,553	36,177	37,262	37,747	37,747	22,253	15,494
AY8	24,851	30,467	35,433	38,586	40,400	41,612	42,153	42,153	15,162	26,991
TOTAL								208,052	149,872	58,180

Table 3.6

Estimated Paid Losses and Loss Reserves by Accident Year,
Based on Mean Paid Loss Development Factors Derived in Table 3.3

Accident Year	Development Year 1	2	3	4	5	6	7	Estimated Ultimate Losses	Paid-To-Date	Estimated Loss Reserve
AY1								14,032	14,032	0
AY2							14,197	14,197	14,015	182
AY3						18,031	18,266	18,266	17,506	760
AY4					22,549	23,226	23,528	23,528	21,599	1,929
AY5				26,019	27,164	27,979	28,343	28,343	23,827	4,516
AY6			25,172	27,488	28,698	29,558	29,943	29,943	21,478	8,465
AY7		27,571	32,314	35,287	36,839	37,944	38,438	38,438	22,253	16,185
AY8	24,487	30,339	35,557	38,828	40,537	41,753	42,296	42,296	15,162	27,134
TOTAL								209,041	149,872	59,169

Table 3.7

Estimated Paid Losses and Loss Reserves by Accident Year, Based on Adjustment to Accident Year AY7

Accident	Development Year							Estimated	Paid-	Estimated
Year	1	2	3	4	5	6	7	Ultimate Losses	To-Date	Loss Reserve
AY1								14,032	14,032	0
AY2							14,197	14,197	14,015	182
AY3						18,031	18,266	18,266	17,506	760
AY4					22,614	23,293	23,595	23,595	21,599	1,996
AY5				25,948	27,167	27,982	28,346	28,346	23,827	4,519
AY6			24,979	27,202	28,481	29,335	29,716	29,716	21,478	8,238
AY7		30,010	34,902	38,008	39,794	40,988	41,521	41,521	24,478	17,043
AY8	25,336	31,062	36,125	39,340	41,189	42,424	42,976	42,976	15,162	27,814
TOTAL								212,649	152,097	60,552

Here is another example of where the chain-ladder method must be used with extreme caution and a lot of professional judgment. Assume that, without informing the actuary, the claims department of the insurer illustrated in Table 3.1 changed its claims settlement philosophy for AY8 so that all losses were paid more promptly. In fact, all of the payments along the calendar year CY8 diagonal increased by 10 percent (except for AY1, development year 7 which has been assumed to be fully mature). This would lead to the following new Table 3.1a.

Table 3.1a

Revised Incremental Loss Payments by Development Year								
	Development Year							
Accident Year	0	1	2	3	4	5	6	7
AY1	5,445	3,157	2,450	1,412	600	352	431	185
AY2	5,847	3,486	1,366	848	1,045	1,054	406	
AY3	5,981	4,854	1,948	2,554	1,680	538		
AY4	7,835	4,453	3,888	3,335	2,296			
AY5	9,763	6,517	3,563	4,382				
AY6	10,745	6,184	5,004					
AY7	14,137	8,928						
AY8	16,678							

This leads to the following new Table 3.2a.

Table 3.2a

Revised Cumulative Loss Payments through Development Years								
	Development Year							
Accident Year	0	1	2	3	4	5	6	7
AY1	5,445	8,602	11,052	12,464	13,064	13,416	13,847	14,032
AY2	5,847	9,333	10,699	11,547	12,592	13,646	14,052	
AY3	5,981	10,835	12,783	15,337	17,017	17,555		
AY4	7,835	22,288	16,176	19,511	21,807			
AY5	9,763	16,280	19,843	24,225				
AY6	10,745	16,929	21,933					
AY7	14,137	23,065						
AY8	16,678							

This in turn leads to the following new Table 3.3a.

Table 3.3a

Revised Age-to-Age Paid Loss Development Factors Based on Cumulative Paym[redacted]							
	Ratio of Successive Development Years						
Accident Year	1/0	2/1	3/2	4/3	5/4	6/5	7/6
AY1	1.580	1.285	1.128	1.048	1.027	1.032	1.013
AY2	1.596	1.146	1.079	1.090	1.084	1.030	
AY3	1.812	1.180	1.200	1.110	1.032		
AY4	1.568	1.316	1.206	1.118			
AY5	1.668	1.219	1.221				
AY6	1.576	1.296					
AY7	1.632						
5-Year Average	1.651	1.231	1.167	1.092	1.048	1.031	1.013

Without reproducing the entire lower triangle, the 5-year average loss development factors and Table 3.3a lead to the following expected values (as compared to Table 3.5).

Table 3.5a

Revised Estimated Paid Losses and Loss Reserves by Accident Year				
Accident Year	**Estimated Ultimate Losses**	**Paid-to-Date**	**Revised Estimated Loss Reserve**	**Table 3.5 Estimated Loss Reserve**
AY1	14,032	14,032	0	0
AY2	14,235	14,052	183	182
AY3	18,334	17,555	779	760
AY4	23,869	21,807	2,062	1,996
AY5	28,954	24,255	4,729	4,519
AY6	30,593	21,933	8,660	8,238
AY7	39,604	23,065	16,539	12,494
AY8	47,279	16,678	30,601	26,991
			63,553	58,180

Thus, in this example, the impact of a decision to settle claims more quickly, which should logically lead to a *smaller* reserve, resulted in a 9.2%

larger reserve using a chain-ladder method applied without the intervention of professional actuarial judgment. Section 3.6.5 explains a method that can be used to overcome this problem.

Finally, this example illustrates that events occurring in any particular calendar year (e.g., a change in reserving philosophy) will be revealed along diagonals. The actuary should take particular care to look for transitions along diagonals, and, if found, should be sure to reconcile such transitions.

The literature contains a number of techniques designed to increase the stability of the chain-ladder method. Some actuaries, for example, discard the highest and lowest loss development factors in each column. Some use a harmonic or geometric mean to dampen the effects of the outliers. Every actuary seems to have a favorite method.

3.6.4 The Bornhuetter Ferguson Method

The reserving actuary will generally calculate the estimated ultimate losses using several methods. Then, by rationalizing differences among the results, the actuary will arrive at a final value. The reserve is then the "best" estimate of the ultimate losses, less total loss payments made to date.

Recall from Section 3.6.2 that the expected loss ratio method can be summarized as

$$\begin{aligned}&\textit{Estimated Ultimate Losses}\\&\qquad = (\textit{Expected Loss Ratio}) \times (\textit{Earned Premium}) \qquad (3.6a)\end{aligned}$$

and

$$\begin{aligned}&\textit{Estimated Loss Reserve}\\&\qquad = (\textit{Estimated Ultimate Losses}) - (\textit{Losses Paid-to-Date}). \qquad (3.6b)\end{aligned}$$

The chain-ladder method can be summarized, for each accident year (i.e., each row of the triangle), as

$$\textit{Estimated Ultimate Losses} = (\textit{Losses Paid-to-Date}) \times \prod_j f_j \qquad (3.7a)$$

where f_j is the loss development factor from a paid-loss-development triangle at duration j (i.e., from $j-1$ to j), and

Estimated Loss Reserve

$$= (\textit{Estimate Ultimate Losses}) - (\textit{Losses Paid-to-Date})$$
$$= (\textit{Losses Paid-to-Date}) \times (f_{ult} - 1), \qquad (3.7b)$$

where $f_{ult} = \prod_j^{\infty} f_j$. Formula (3.7b) can also be written as

Estimated Loss Reserve

$$= (\textit{Estimated Ultimate Losses}) \times \left(1 - \frac{1}{f_{ult}}\right). \qquad (3.7c)$$

The ***Bornhuetter Ferguson method*** formally combines the expected loss ratio and chain-ladder methods of loss reserving as just described. The Bornhuetter Ferguson method is meant to be a stabilizer for long-tail lines or immature data. It only uses initial loss ratio expectations to the extent that losses are not paid or reported. In addition, it assumes that past experience is not fully representative of the future.

For each accident year row of the loss development triangle (e.g., Table 3.2), the actuary estimates the ultimate loss ratio expected for that accident year, taking into consideration all available information including losses paid-to-date and data used for pricing. Using this estimated loss ratio and the earned premium for that accident year, the actuary calculates the ***estimated ultimate total losses*** for that accident year. The actuary then uses the paid age-to-age loss development factors derived by the chain-ladder method (e.g., Table 3.3) to calculate age-to-ultimate factors that can be used to develop immature losses paid-to-date to their fully mature ultimate level.

The Bornhuetter Ferguson method can be summarized algebraically by the following steps. From the expected loss ratio method for each accident year we derive

Estimated Ultimate Losses

$$= (\textit{Expected Loss Ratio}) \times (\textit{Earned Premium}). \qquad (3.6a)$$

We can then use this value of estimated ultimate losses, along with the loss development factors from the chain-ladder method and formula (3.7c), to calculate

$$\textit{Estimated Loss Reserve} = (\textit{Estimated Ultimate Losses}) \times \left(1 - \frac{1}{f_{ult}}\right). \qquad (3.8)$$

In this way, we arrive at the actuarial reserve using the Bornhuetter Ferguson method. An example will make the process clearer.

EXAMPLE 3.1

You have chosen the following paid loss development factors to model the lower half of a claims paid rectangle:

1/0	2/1	3/2	4/3	∞/4
1.41	1.22	1.16	1.08	1.04

You are setting reserves for the annual report as of December 31, CY7. For AY6 you have claims paid-to-date of $420,000 at report duration 1, which is December 31, CY7. The earned premium calculated for AY6 is $1,000,000 and the expected loss ratio is .600. For AY6, determine the estimated loss reserve using each of the expected loss ratio method, the chain-ladder method, and the Bornhuetter Ferguson method.

Solution

Expected Loss Ratio Method

Estimated Ultimate Losses

$$= \text{(Expected Loss Ratio)(Earned Premium)}$$
$$= (.600)(1{,}000{,}000) = 600{,}000.$$

Since the losses paid-to-date amount is 420,000, then the estimated loss reserve is

$$600{,}000 - 420{,}000 = 180{,}000.$$

Chain-Ladder Method

Estimated Ultimate Losses

$$= \text{(Losses Paid-to-Date)} \times \left(\prod_j f_j\right)$$
$$= (420{,}000)(1.22 \times 1.16 \times 1.08 \times 1.04)$$
$$\approx (420{,}000)(1.59) = 667{,}800.$$

Note: The factor $f = 1.41$ is not used since for AY6, we are at report duration 1.

Again, the claims paid-to-date amount is 420,000, so the estimated loss reserve is

$$667{,}800 - 420{,}000 = 247{,}800.$$

Bornhuetter Ferguson Method

From the expected loss ratio method we have estimated ultimate losses of 600,000, and from the chain-ladder method we have $f_{ult} = \prod_j f_j = 1.59.$

Then the Bornhuetter Ferguson method produces

$$
\begin{aligned}
\text{Estimated Loss Reserve} & \\
&= \text{Expected Losses to be Paid in the Future} \\
&= 600{,}000\left(1-\frac{1}{f_{ult}}\right) \\
&= 600{,}000\left(1-\frac{1}{1.59}\right) = 222{,}642.
\end{aligned}
$$

Note that if the expected loss ratio had been .6678, then both the loss ratio method and the chain-ladder method would have produced estimated ultimate losses of 667,800 and an estimated loss reserve of 247,800. Then the Bornhuetter Ferguson method would produce

$$\text{Estimated Loss Reserve} = 667{,}800\left(1-\frac{1}{1.59}\right) = 247{,}800,$$

as expected. □

This is the danger of using the expected loss ratio produced from the chain-ladder method in the Bornhuetter Ferguson approach. It may appear that two methods produce the same answer, but in reality only one method was used.

The Bornhuetter Ferguson method has the advantages of (a) being more stable than the chain-ladder method, and (b) allowing inclusion of data from other sources (e.g., in arriving at an expected loss ratio). Its major disadvantage is that it requires an assumption, perhaps from an outside data source, for the initial expected loss ratio, and such data may not be available.

EXAMPLE 3.2

Given:

Earned Premium $=$ \$800,000
$E[LR] = .680$
Σ Individual Claim File Estimates (\$Paid + Reserves) $=$ \$500,000
\$Paid-to-date $=$ \$300,000
$\prod f_j$ (on incurred claims) $= 1.100$

Find the Total Actuarial Reserve using:

(a) The Chain Ladder Method;
(b) The Loss Ratio Method;
(c) The Bornhuetter Ferguson Method.

Solution

a) The Chain Ladder Method:

$$
\begin{array}{lcr}
Est[ULT\$L] = \$500{,}000 \times 1.100 & = & \$550{,}000 \\
\$\text{ paid-to-date} & = & \underline{300{,}000} \\
\text{Total Actuarial Reserve} & = & 250{,}000
\end{array}
$$

b) The Loss Ratio Method:

$$
\begin{array}{lcr}
Est[ULT\$L] = \$800{,}000 \times 0.680 & = & \$544{,}000 \\
\$\text{ paid-to-date} & = & \underline{300{,}000} \\
\text{Total Actuarial Reserve} & = & 244{,}000
\end{array}
$$

(c) The Bornhuetter Ferguson Method:

$$Est[ULT\$L] = 800{,}000(0.680) = \$544{,}000$$

$$
\begin{array}{lcllr}
E[IBNR \text{ in Incurreds}] & = & (544{,}000\left(1-\dfrac{1}{f_{ULT}}\right) & & \\
 & = & 544{,}000\left(1-\dfrac{1}{1.10}\right) & = & 49{,}455
\end{array}
$$

Plus

$$
\begin{array}{lcllr}
\text{Reserves in Case Files} & = & 500{,}000-300{,}000 & = & \underline{200{,}000} \\
\text{Total Actuarial Reserve} & & & = & 249{,}455
\end{array}
$$

□

EXAMPLE 3.3

Prove that:

$$R_{BF} = \left(1-\frac{1}{f}\right)\times R_{LR}+\left(\frac{1}{f}\times R_{CL}\right)$$

where:

R_{BF} is the Loss Reserve under the Bornhuetter Ferguson method;

R_{LR} is the Loss Reserve under the Loss Ratio method;

R_{CL} is the Loss Reserve under the Chain Ladder method.

and f is Πf_j or the product of the Chain Ladder link ratios from now to full maturity.

Proof:

$$\begin{aligned}
R_{BF} &= E[ULT\$L]_{LR}(1-1/f) \\
&= (1-1/f)E[LR]\times(\text{Earned Premium}) \\
&= (1-1/f)E[LR]\times(EP)-(1-1/f)(\text{Paid-to-Date}) \\
&\qquad\qquad +(1-1/f)(\text{Paid-to-Date}) \\
&= (1-1/f)\left[E[LR]\times(EP)-(\text{Paid-to-Date})\right] \\
&\qquad\qquad +(1/f)[(\text{Paid-to-Date})\times f-(\text{Paid-to-Date})] \\
&= (1-1/f)\left[E[LR]\times(EP)-(\text{Paid-to-Date})\right] \\
&\qquad\qquad +(1/f)\left\{E\left[ULT\$L\right]_{CL}-(\text{Paid-to-Date})\right\} \\
&= (1-1/f)R_{LR}+(1/f)R_{CL}
\end{aligned}$$

❒

3.6.5 ESTIMATES SPLIT INTO FREQUENCY AND SEVERITY

Depending on the availability of accurate data, this method can provide the actuary with very valuable information. The loss reserve analysis is actually based on two development triangles, one for the ***incurred claim count*** (i.e., number of reported claims), and one that would be called ***average payment per claim incurred*** (if derived from

paid dollars) ***or average incurred per claim incurred*** (if derived from incurred dollars).

The claim count triangle is generally quite dependable, matures rapidly to its ultimate values, and can usually be used with confidence. Thus this frequency component provides a triangle of data that can be analyzed for meaningful trends and, therefore, can be projected to ultimate with a high level of confidence. To illustrate, consider the following data.

Table 3.8

Cumulative Loss Payments (000) through Development Years					
Accident Year	Development Year				
	0	1	2	3	4
AY4	1,200	4,500	7,000	8,800	10,000
AY5	1,600	5,100	8,800	12,500	
AY6	1,700	6,800	11,000		
AY7	2,080	7,600			
AY8	2,533				

Assume we also know the cumulative number of claims closed each year for each accident year as is shown below:

Table 3.9

Cumulative Closed Claims through Development Year						
Accident Year	Development Years					Estimated Ultimate Number of Claims
	0	1	2	3	4	
AY4	400	700	850	950	1,000	1,000
AY5	480	790	975	1,130		1,200
AY6	500	950	1,190			1,400
AY7	570	1,050				1,500
AY8	600					1,500

The ultimate number of claims closed for each accident year has been projected from the incurred claim count triangle. If we divide the cumulative loss payments at each maturity by the corresponding number of claims closed, the following average claim size triangle is calculated.

Table 3.10

Cumulative Average Claim Size through Development Year						
Accident Year	Development Years					Estimated Ultimate Number of Claims
	0	1	2	3	4	
AY4	3,000	6,429	8,235	9,263	10,000	1,000
AY5	3,333	6,456	9,026	11,062		1,200
AY6	3,400	7,158	9,244			1,400
AY7	3,649	7,238				1,500
AY8	4,222					1,500

We could now proceed to establish the estimated ultimate payment per claim. For each accident year, we would then multiply the estimated ultimate number of claims to be closed by the average ultimate average claim size. For example since all claims for AY4 are closed, we will assume that full maturity is at Year 4. Then, multiplying 1000 claims by $10,000 per claim produces ultimate losses of $10,000,000 for AY4. Of course for later years, the ultimate average claim size will first be projected to development year 4.

A second frequency and severity approach to estimating ultimate paid losses is the ***closure method***. This method analyzes the rate of the emergence of claim counts, the rate of claims settlement, and the rate of growth in loss payments.

With the closure method, the loss reserve is determined as the sum of the projected incremental paid losses for each future development year. The projected incremental paid losses are determined by multiplying the projected paid severity by the estimated number of claims to be closed at each development year. We analyze the proportion of closed claims triangle to estimate the number of claims that will be closed at each development year.

Example 3.4 as illustrated here shows one application of this method. In practice, actuaries use varying assumptions through the different steps of this approach resulting in the estimated ultimate paid losses of claim counts (frequency) multiplied by loss dollars per claim (severity).

EXAMPLE 3.4

You are given the following information:

From Table 3.8 we calculate the incremental loss payments triangle. The 3,300 for AY4, maturity 1 is found by subtracting the maturity 0 entry in Table 3.8 (1,200) from the maturity 1 entry in that line (4,500).

Table 3.11

Incremental Loss Payments (000) through Development Year					
Accident Year	Development Years				
	0	1	2	3	4
AY4	1,200	3,300	2,500	1,800	1,200
AY5	1,600	3,500	3,700	3,700	
AY6	1,700	5,100	4,200		
AY7	2,080	5,520			
AY8	2,533				

From Table 3.9 we similarly calculate the incremental closed claims triangle.

Table 3.12

Incremental Closed Claims through Development Year					
Accident Year	Development Years				
	0	1	2	3	4
AY4	400	300	150	100	50
AY5	480	310	185	155	
AY6	500	450	240		
AY7	570	480			
AY8	600				

Assume the annual severity trend is 5%.

(a) Project incremental paid severity through development year 4 for each accident year (round to 2 decimal places).
(b) Project incremental closed claims through development year 4 for each accident year (round to the nearest whole number).
(c) Calculate the estimated total reserve for all accident years using the results from parts (a) and (b).

Solution

(a) Step 1: Calculate the incremental average loss payments (severity) triangle (Table 3.11 / Table 3.12).

Table 3.13

Incremental Severity (000) through Development Year					
Accident Year	Development Years				
	0	1	2	3	4
AY4	3.00	11.00	16.67	18.00	24.00
AY5	3.33	11.29	20.00	23.87	
AY6	3.40	11.33*	17.50		
AY7	3.65	11.50			
AY8	4.22				

* $11.33 = \frac{5,100}{450}$

Step 2: Adjust incremental severity to AY8 levels and use the average as the selected severity for each development year.

Table 3.14

Incremental Severity (000) at AY8 Level through Development Year					
Accident Year	Development Years				
	0	1	2	3	4
AY4	3.65	13.37	20.26	21.88	29.17
AY5	3.86	13.07	23.15	27.63	
AY6	3.75	12.50*	19.29		
AY7	3.83	12.08			
AY8	4.22				
Average	3.86	12.75	20.90	24.76	29.17

* $12.50 = 11.33 \times 1.05^2$

Step 3: Complete the bottom half of Table 3.13. For development Years 1-4, start with the selected severity from Step 2 and detrend by 5% to complete the bottom half.

Table 3.13a

Incremental Severity (000) through Development Year					
Accident Year	Development Years				
	0	1	2	3	4
AY4	3.00	11.00	16.67	18.00	24.00
AY5	3.33	11.29	20.00	23.87	*25.20*
AY6	3.40	11.33	17.50	*22.45*	*26.46*
AY7	3.65	11.50	*19.91*	*23.58**	*27.78*
AY8	4.22	*12.75*	*20.90*	*24.76*	*29.17*

* $23.58 = 24.76 \div 1.05$

(b) Step 4: Calculate the percentage of claims closed triangle. The percentage of closed claims is determined by calculating the number of closed claims as a percentage of the number of claims that remain open. Use the ultimate claim counts in Table 3.9.

Table 3.15

Percentage of Claims Closed through Development Year					
Accident Year	Development Year				
	0	1	2	3	4
AY4	40.0%	50.0%	50.0%	66.7%	100.0%
AY5	40.0%	43.1%	45.1%	68.9%	
AY6	35.7%	50.0%	53.3%		
AY7	38.0%	51.6%*			
AY8	40.0%				
Average	38.7%	48.7%	49.5%	67.8%	100.0%

* $51.6\% = \dfrac{480}{1{,}500 - 570}$

Step 5: Complete the bottom half of Table 3.12. Use the ultimate claim counts from Table 3.9. Then subtract the closed counts to yield open claims, then multiply by the selected percentage from Table 3.15.

Table 3.12a

Incremental Closed Claims through Development Year					
Accident Year	Development Years				
	0	1	2	3	4
AY4	400	300	150	100	50
AY5	480	310	185	155	*70*
AY6	500	450	240	*142*	*68*
AY7	570	480	*223**	*154*	*73*
AY8	600	*438*	*229*	*158*	*75*

* $223 = 49.5\% \times (930 - 480)$

(c) Total reserve:

Table 3.16

Projected Incremental Loss Payments (000) through Development Year					
Accident Year	Development Years				
	1	2	3	4	Reserve
AY5				1,764	1,764
AY6			3,188	1,799	4,987
AY7		4,440	3,631*	2,028	10,099
AY8	5,585	4,786	3,912	2,188	16,470
Total					33,321

* $3{,}631 = 154 \times 23.58$ ❑

Note that to do this accurately the actuary should split total claim dollars paid into those paid on closed files versus those partial payments made to date on files that are still open. The actuary should also be careful when counting claims and using them for averages, since the claim count should exclude claims closed with no payment. If data on number of claims closed with no payment is not available, then it is important to determine that the claim count data have been defined consistently over the years.

Additional insights can be provided by reviewing Table 3.13. Table 3.13 is repeated here in dollars rather than thousands of dollars.

Table 3.13

Incremental Severity through Development Year					
Accident Year	Development Years				
	0	1	2	3	4
AY4	3,000	11,000	16,667	18,000	24,000
AY5	3,333	11,290	20,000	23,871	
AY6	3,400	11,333	17,500		
AY7	3,649	11,500			
AY8	4,222				

It is seen in Table 3.13 that at each greater maturity age, the average closed claim size (severity) increases. This is expected, since the more difficult and expensive cases take the longest to settle. Also, at each age, claims in each accident year tend to cost more than in the prior year (i.e. comparing successive values down each column). This is due to inflation. The one exception to the latter observation is Development Year 2 for AY5. This may prompt some investigation. Perhaps there was one very large claim settled in that period for which some adjustment may be appropriate.

3.6.6 Summary

This section has presented a number of loss reserving methods. Without reviewing the context in which the actuary is forced to work, it is impossible to say that any one method is better than another. Normally the actuary will determine an ultimate loss estimate using more than one method and then calculate a reserve estimate by subtracting losses paid-to-date. The process of reconciling differences in the results is often the key to achieving a confident final reserve estimate.

The actuary would need to consult the particular standards of practice that govern their work, in the selection of the final reserve estimate. The standards of practice may refer to a best estimate or a central estimate.

3.7 Discounting Loss Reserves

The issue of ***discounting*** loss reserve estimates to reflect the time value of future loss payments has been a surprisingly controversial actuarial topic for the property/casualty actuary. In contrast, for the life actuary, discounting has been used since the issuance of the first life insurance policy. Obviously we need not set aside \$1 today for a \$1 loss payment expected several years later. It is a basic actuarial principle that we should account for the ***time value of money***. Under statutory accounting regulations in many jurisdictions, however, property/casualty reserves must be carried at their full undiscounted value. The one major exception is that the tabular reserves for workers compensation may be discounted. These are annuity-type wage replacement or survivor benefits that are paid on a regular basis over an extended period of time. In the United States, the allowable discount rate is prescribed, as is the mortality table to be used. There are published tables, which can be consulted to set the reserves on each such claim.

A reason given for using undiscounted loss reserves is that non-discounting provides a level of conservatism that is fiscally prudent. In fact, in some jurisdictions, the difference between undiscounted and discounted reserves is viewed as an *implicit* provision for adverse deviation.

There seems to be growing agreement, however, that an *explicit* provision for adverse deviation is preferable, although in the United States there is not yet a consensus on provision methodology. Until an explicit provision for adverse deviation is adopted, the use of undiscounted reserves to provide an implicit provision will probably continue.

Discounting loss reserves is not conceptually difficult. If we take the run-off (or future payment) pattern assumed in Table 3.6, using the mean to establish the model parameters, we can establish the lower triangle representing expected future loss payments shown in Table 3.17. These expected future loss payments can be discounted to reflect the time value of money, and used to calculate a discounted loss reserve estimate. Assume that all payments are made at the midpoint of each development year, that development year 7 is the last development year (i.e., no payments are made after that year), and that the effective annual rate of interest earned by the assets backing the reserves is 5%. Discounting the expected cash flows in Table 3.17 produces Table 3.18, with a total discounted loss reserve estimate of 54,238. For example, for AY6, development year 4, $2{,}316(1.05)^{-1.5} = 2{,}153$, and so on.

Table 3.17

Expected Future Loss Payments by Accident and Development Years, Based on Table 3.6								
Accident Year	0	1	2	3	4	5	6	7
AY1								
AY2								182
AY3							525	234
AY4						950	676	302
AY5					2,192	1,145	815	364
AY6				3,694	2,316	1,209	861	384
AY7			5,318	4,742	2,973	1,553	1,105	493
AY8		9,325	5,852	5,218	3,271	1,708	1,216	543

Table 3.18

Discounted Values (as of December 31, AY8) of Expected Future Loss Payments by Accident and Development Years, Based on Table 3.6 and 5% Interest									
Accident Year	Development Years								Discounted Loss Reserve
	0	1	2	3	4	5	6	7	
AY1									
AY2								178	178
AY3							512	217	730
AY4						927	628	267	1,823
AY5					2,139	1,064	721	307	4,232
AY6				3,605	2,153	1,070	726	308	7,862
AY7			5,190	4,407	2,632	1,309	887	377	14,802
AY8		9,100	5,439	4,619	2,758	1,371	930	395	24,612
									54,238

The above illustration assumes that the loss reserve has been determined from a paid-loss-development triangle. Other methods of loss reserving are possible. If the loss reserve is not established using a paid-loss-development triangle, it should still be possible to allocate the loss reserve (which is equal to expected future loss payments) in proportion to the pattern presented by an appropriate paid-loss-development triangle, which would allow the calculation of tables similar to Tables 3.17 and 3.18.

An actuary using the discounted estimate as the annual statement loss reserve liability should also explicitly allow an appropriate provision for adverse deviation. Note that by using non-discounted reserves, an implicit margin for contingencies, equal to the difference between the non-discounted reserve and the discounted reserve, has been established.

Although the discounting methodology is conceptually simple, there are several issues that must be addressed in practice. The illustration assumes that all payments are made at the midpoint of the respective development year. That assumption should be tested and modified if necessary. For most lines of business, we would expect payments in development year 1 to be skewed to an earlier average date than a midpoint assumption. Furthermore, the illustration assumed that the payments in development year 7 completed the development pattern. It would be natural to have the last entry be a small reserve estimate. This would lead to a final age-to-ultimate development factor appropriately represented by $\infty / 7$. A special analysis would be required to establish an average date for the final payments made after development year 7.

Another issue is the choice of an interest rate to be used in the discounting process. Conceptually, this interest rate should be the one that will be earned by the assets held to fund the reserve cash flow requirements until loss payments are finalized. It may not, however, be clear exactly which assets are backing those liabilities (unless the portfolio is perfectly matched, which is uncommon for a property/casualty insurer). Further, given a block of assets, questions arise as to the expenses associated with the investment income, and the inclusion of unrealized capital gains.

A considerable degree of tax planning is also involved, but a discussion of this is beyond the scope of this text.

Finally, the concept of discounted cash flows will be introduced in the Chapter Four methodologies for estimating loss reserves and discounting. The establishment of loss reserves is an integral part of the establishment of rates, since rates are based on future incurred claims developed to ultimate for the policy period the rates will be in effect. In theory, the loss dollars entering the ratemaking calculation should be discounted cash flows. One difference between the discounting calculations of the pricing actuary and those of the loss reserving actuary is the date of valuation. The reserve actuary generally for financial statement purposes will be using December

31 as the date of valuation. The pricing actuary will discount cash flows to the average date the rate will be in effect as the date of valuation in the pricing analysis.

In the United States, regulators generally do not accept discounting of loss reserves on insurers' financial statements, while they do require that discounted cash flows be used in pricing. They argue that the balance sheet loss reserves should be on a conservative basis. Loss reserves are discounted for Federal Income Tax purposes.

3.8 EXERCISES

Section 3.3.3

3.1 Describe the difference between gross IBNR and pure IBNR.

Section 3.3.5

3.2 An actuary recently evaluated a company's loss reserves, which were based on both paid loss development and incurred loss development (net of reinsurance). The company recently increased its retention limit from 75,000 to 150,000. Which method would you adopt as more appropriate for estimating the actuarial reserve liability (net of reinsurance)? Why?

Section 3.3.6

3.3 Define salvage and subrogation, and outline their impact on the loss reserving process.

Section 3.6.2

3.4 Briefly describe how you would determine an estimated loss reserve for a new line of business using the expected loss ratio approach.

Section 3.6.3

3.5 You are given the following information on cumulative incurred losses through the development years shown.

Accident Year	Development Year 0	1	2	3	4	Paid-to-Date as of 12/31/AY7
AY1	200.0	250.0	275.0	297.5	300.0	300.0
AY2	230.0	287.5	312.0	339.4	345.6	345.6
AY3	264.5	324.0	354.0	387.4	396.8	396.8
AY4	296.6	365.4	401.9	442.2		380.2
AY5	332.7	412.4	456.7			349.8
AY6	373.6	465.5				301.7
AY7	419.7					231.3

Estimate the loss reserve as of December 31, AY7, using each of the following models. In each case, split the total loss reserve into gross IBNR and case reserves.

(a) An average factor model
(b) A 4-year average factor model
(c) A mean factor model

3.6 The following data represent cumulative loss payments made through the development years shown.

Accident Year	Development Year 0	1	2	3	4	5	6
AY1	1780	2673	2874	3094	3157	3166	3166
AY2	3226	4219	4532	4881	5144	5199	
AY3	3652	4989	5762	6436	6720		
AY4	2723	4301	5526	6231			
AY5	2923	4666	5349				
AY6	2990	5417					
AY7	3917						

Estimate the loss reserve as of December 31, AY7, using each of the following.

(a) An average factor model
(b) A 5-year average factor model
(c) A mean factor model

Section 3.6.4

3.7 You have the following Earned Premiums, Expected Loss Ratios, and Paid Claims data:

Year	Earned Premium ($,000)	Expected Loss Ratio
AY4	4,750	.60
AY5	5,175	.62
AY6	5,500	.65
AY7	5,900	.63

Cumulative Paid Claims ($,000)				
	Development Year			
Acc Year	0	1	2	3
AY4	2,147	2,697	2,843	2,843
AY5	2,312	3,091	3,106	
AY6	2,451	3,142		
AY7	2,612			

Calculate the reserve liability (undiscounted) as of December 31, AY7 using the Bornhuetter Ferguson Method. Use the mean factor model with no development past Development Year 2.

3.8 You are given the following information for a certain accident year.

Earned Premium:	1000
Expected Loss Ratio:	.650
Paid Loss Factor f_{ult} :	1.21
Incurred-to-Date:	600
Paid-to-Date:	500

Find the estimated loss reserve using the Bornhuetter Ferguson method.

3.9 You are given the following information on losses paid during each of AY4 through AY7.

Accident Year	Earned Premium	2004	2005	2006	2007
AY4	25,000	10,000	5,000	2,000	0
AY5	29,750		12,050	6,025	2,400
AY6	33,000			14,500	7,250
AY7	38,000				17,475

The expected loss ratios are as follows:

Accident Year	$E[LR]$
AY4	0.680
AY5	0.688
AY6	0.700
AY7	0.700

Find the December 31, AY7 estimated loss reserve using each of the following methods.

(a) Expected loss ratio method
(b) Chain-ladder method (round all factors to 3 decimals)
(c) Bornhuetter Ferguson method

Section 3.6.5

3.10 For the data of Exercise 3.13, assume you have the following additional information showing the cumulative number of claims through development years.

Accident Year	0	1	2	3	4	5
AY2	4962	5583	5893	6141	6203	6203
AY3	5098	5735	6052	6307	6372	
AY4	5204	5855	6180	6440		
AY5	5210	5861	6186			
AY6	6018	6771				
AY7	6600					

Calculate an alternate loss reserve estimate, as suggested in Section 3.6.5, using an all-years average method.

3.11 The following data showing incremental loss payments (in thousands) and incremental closed claims are given.

Incremental Loss Payments (000) through Development Year					
Accident	Development Years				
Year	0	1	2	3	4
AY4	2,000	4,000	3,000	2,200	2,800
AY5	2,600	4,240	4,080	4,680	
AY6	2,380	6,580	5,440		
AY7	3,120	7,680			
AY8	3,800				

Incremental Closed Claims through Development Year						
	Development Years					Ultimate
Accident Year	0	1	2	3	4	Claim Counts
AY4	400	300	150	80	70	1,000
AY5	480	310	210	140		1,200
AY6	500	450	240			1,400
AY7	570	480				1,500
AY8	600					1,500

Assuming the annual severity trend is 5%, calculate the loss reserve using the claims closure approach as described in Section 3.6.5.

Section 3.7

3.12 For the results in Exercise 3.6, calculate the discounted loss reserve in each case, assuming $i = .07$ and all payments made at midyear.

3.13 You are given the following data on incremental accident year loss payments (in thousands) by development year.

Accident Year	Development Year					
	0	1	2	3	4	5
AY2	192	251	153	145	98	0
AY3	205	280	195	150	102	
AY4	230	345	230	212		
AY5	288	410	275			
AY6	398	563				
AY7	530					

(a) Using the chain-ladder method based on both average loss development factors and mean loss development factors, calculate the loss reserve as of December 31, AY7.

(b) Determine the discounted loss reserve using $i = .05$. Assume all payments are made at midyear.

3.14 Your company issues a general liability insurance policy on 1/1/AY7. Expected losses on the policy are 16,000,000 with the following expected payment pattern.

Months from Beginning of Accident Year	Cumulative Percentage Paid
12	20%
24	45
36	60
48	75
60	90
72	100

Consistent with this, 3,200,000 has been paid as of December 31, AY7. Assuming all future loss payments are made in the middle of each year, calculate the discounted loss reserve to be carried by the company as of December 31, AY7 using $i = .04$.

3.15 An actuary, using a combination of methods, has decided that the following are appropriate reserves by accident year.

Accident Year	Adopted Loss Reserve
AY1	0
AY2	0
AY3	48
AY4	298
AY5	822
AY6	1,765
AY7	4,008
Total	6,941

The actuary also has the following data representing cumulative loss payments through the development years shown.

Accident Year	Development Year						
	0	1	2	3	4	5	6
AY1	1,780	2,673	2,874	3,094	3,157	3,166	3,166
AY2	3,226	4,219	4,532	4,881	5,144	5,199	
AY3	3,652	4,989	5,762	6,436	6,720		
AY4	2,723	4,301	5,526	6,231			
AY5	2,923	4,666	5,349				
AY6	2,990	5,417					
AY7	3,917						

Using the mean loss development factors, develop the future paid losses by development year (see Exercise 3.6(c)). Then, for each accident year, allocate the adopted loss reserve for that payment year, using the future paid loss triangle. Finally, assuming all payments are made at midyear, calculate the discounted loss reserve using $i = 6.5\%$.

3.16 Is it possible for discounted loss reserves to exceed undiscounted loss reserves? Discuss.

3.17 You are given the following information:

Incremental Severity at AY5 Level through Development Year					
Accident Year	Development Years				
	0	1	2	3	4
AY5	5,600	7,300	9,400	11,200	16,500

Cumulative Closed Claims through Development Year						
Accident Year	Development Years					Estimated Ultimate Number of Claims
	0	1	2	3	4	
AY1	310	500	655	795	840	840
AY2	330	535	705	860		910
AY3	360	575	760			975
AY4	350	570				965
AY5	410					1,120

Percentage of Claims Closed through Development Year					
Accident Year	Development Year				
	0	1	2	3	4
Selected	37.0%	36.0%	46.0%	75.0%	100.0%

(a) Calculate the total reserve using the closure method and assuming the annual severity trend is 7%.

(b) Calculate the discounted reserve assuming payments are made at midyear and an annual rate of interest of 4%.

3.18 You are given the following data:

Accident Year	Incurred Claims ($000)			
	Development Year			
	0	1	2	3
AY4	2,147	2,202	2,214	2,214
AY5	2,312	2,390	2,402	
AY6	2,451	2,520		
AY7	2,612			

(a) How does an Incurred-Claims Triangle differ from a Paid-Claims Triangle?

(b) Is it possible for incurred claims to decrease from development year t to $t+1$? If your answer is yes, explain how this could happen.

(c) Under normal circumstances, would you expect larger or smaller loss development factors in a "paid" triangle versus an "incurred" triangle?

(d) Using an "average loss development factor" method, estimate the total Incurred IBNR Liability as at December 31, AY7. (Carry 4 decimals.)

(e) Determine the Discounted Incurred IBNR Liability as at December 31, AY7 using $i=7\%$. (State any necessary assumptions.)

3.19 You are given the following information:

Earned Expenses	Accident Year	Cumulative Paid Losses ($000's) at Development Year 0	1	2	3	4
100,000	AY3	200	240	260	270	270
100,000	AY4	165	235	255	265	
100,000	AY5	205	245	265		
100,000	AY6	195	235			
100,000	AY7	203				

(a) Describe a highly probable error in the data. If you assume there is an error, what would you do?

(b) Given the adjustment you made in part (a), now determine your best estimate of the loss reserve required at December 31, AY7 using "mean" Loss Development Factors.

(c) Determine the discounted value of the loss reserves if $i = 5\%$ per annum effective. State all assumptions.

3.20 Assume that accident year losses are paid out in the following pattern:

1-12 months	40%
13-24 months	30%
25-36 months	15%
37-48 months	10%
49-60 months	5%

In accident year AY3, ultimate incurred losses were $1,000,000. Since then economic inflation has averaged 5% per annum effective and your book of business has grown 3% per annum effective. What loss reserve should be carried at December 31, AY7?

RATEMAKING o 4

4.1 INTRODUCTION

Chapter Two introduced the reader to several of the more widely-sold property/casualty coverages, using a description that was very general. The intent was to discuss the types of economic security provided by property/casualty insurers around the world without describing any particular coverage offered by any particular insurer.

Chapter Three introduced the reader to various methods the actuary uses to determine loss reserves. In determining loss reserves, the actuary must first estimate the ultimate losses, and this estimate is a fundamental step in property/casualty ratemaking.

Chapter Four provides an introduction to property/casualty ratemaking without telling the reader how to develop rates for any particular coverage or any particular jurisdiction. Instead, this chapter introduces the essential ingredients inherent in almost all property/casualty ratemaking. Foundations and principles are discussed rather than particular methodologies applicable to particular problems. Although the methods explored have broad application, the reader is not prepared to set rates for any particular coverage without additional study. Ratemaking methodologies for specific types of property/casualty insurance coverages are presented in the advanced syllabus of various actuarial educational organizations.

4.2 OBJECTIVES OF RATEMAKING

The following ratemaking objectives are subdivided into those that are essential and those that are highly desirable but not essential.

4.2.1 ESSENTIAL OBJECTIVES

Cover Expected Losses and Expenses
Obviously for the insurer to stay in business, income must at least equal outgo. Income includes premium and investment income, and outgo includes all losses, the expenses associated with those losses, all sales expenses (e.g.,

commissions), premium taxes, and all expenses of running the head office and any branch offices. Each cohort of policyholders should pay for its expected costs. There should not be any "inter-generational" subsidies, nor should there be any subsidies among risk classes. That is, each risk class should pay a premium commensurate with the risk that it contributes to the insurance pool. This requires the pricing actuary to estimate the costs that will be incurred within the risk class through the payment of the last claim dollar, which may not occur for many years. Methods for making such estimates will be outlined later in this book. It is possible for management to decide to sell some products below cost. If this decision is made, then the expected loss from such a marketing decision should come from the owner's equity (surplus) account and not from any other group of policyholders.

Produce Rates That Make Adequate Provision for Contingencies

Not only must the actuary price for the expected, but there should also be a provision in the rates for the unexpected (e.g., the 100-year flood). This is not an easy matter. Property/casualty insurance is extremely competitive with dozens of insurers vying for consumers' dollars. Further, in many lines, the consumer has the ultimate prerogative to self-insure. So if rates are too high, the insurer will lose business, and normally its best business, to the competition or to self-insurance. If rates are too low, however, the insurer will lose money and be forced to reduce surplus to cover the deficiency. This, in turn, reduces the opportunity to expand the number of policies underwritten in the future, since surplus is needed to support the writing of new business. Inadequate rates also make senior management, shareholders, and, ultimately, regulators very unhappy since they endanger the insurer's solvency.

Encourage Loss Control

A well-designed risk classification process will provide strong economic incentives for the policyholder to reduce loss costs by reducing claim frequency, or loss severity, or both. Examples of methods used to encourage loss control are good driver discounts in auto insurance, discounts for sprinkler systems and burglar alarms in homeowners and commercial property insurance, discounts for accident prevention and rehabilitation programs in workers compensation, and so on. Encouraging loss control not only allows the insurer to offer lower rates, it also provides an important service to society by reducing accidents, and the injuries and property damage that ensue. Hence, there is a very real social value to this aspect of the ratemaking process.

Satisfy Rate Regulators
Nearly all states and provinces have insurance laws and departments or agencies that regulate, or at least review, insurance rates for lines such as automobile and workers compensation. In general terms, the basic requirements of the regulatory agencies are that rates must be adequate, not excessive, and not *unfairly* discriminating.

Most agencies will ask the actuary to support a proposed rate change with full actuarial documentation. Many agencies employ staff and/or consulting actuaries to review rate filings. There may also be a public hearing at which the actuary must appear to defend the proposal under cross examination by lawyers representing the regulatory agency and possibly insurance consumer groups.

The agency will usually criticize and even refuse methods that appear to be subjective and/or capricious. Thus the actuary is well advised to use generally accepted actuarial techniques in the analysis. These techniques are outlined in detail later in this chapter.

The actuary must also be prepared to defend methods that are widely accepted in the actuarial community, but may appear to be socially unacceptable. Examples include the classification of risks using variables such as age, gender, and marital status. There may also be a desire on the part of the agency, especially if its head is elected, to make rates "affordable." Not only must the rating methodology be actuarially defensible, but the actuary must have the ability to convince a lay audience of that fact.

4.2.2 Non-Essential but Desirable Objectives

Produce Rates That Are Reasonably Stable
If rates were to rise and fall for reasons not understood by the policyholders, a justifiable level of skepticism would be created among consumers. Thus, the effect on rates of one big storm or flood is spread out over long periods. The use of reinsurance helps spread the effects of losses over time, thus enhancing rate stability.

Produce Rates That Are Reasonably Responsive to Changes
Rates should respond reasonably quickly to changes that have immediate and long-term consequences. A legislated change in the speed limit, for example, or an important precedent set by a Supreme Court decision should be responded to as quickly and fully as possible.

This objective is somewhat contradictory to the previous one. Methods that might be used to enhance stability could dampen responsiveness, and methods that would enhance responsiveness might override stability. This requires a delicate actuarial compromise, always entailing human judgment. There are statistical tools, such as loss-development factors and trend factors, that assist the actuary in finding the right blend of stability and responsiveness, and they will be discussed later in this chapter. The most important of these is credibility theory.

Be Simple and Easy to Understand

It will be difficult to promote loss control if the policyholder does not understand the connection between loss control and lower rates. Agents and brokers must be able to work with the insurer's rate manual. If the manual is too complex it may provide an incentive for agents and brokers to place the business with another insurer. Complex systems are more expensive to program and maintain in a computer. You may wish, for example, to use eight variables to classify risks for auto insurance, but this may not be acceptable if the rest of the industry is only using four. Finally, the actuary will have to "sell" the rating scheme to senior management and often to regulators, who may not have strong analytic skills. It is wise, therefore, to produce a methodology that is understandable to a non-technical person.

4.3 Data for Ratemaking

The ratemaking actuary will need access to data on loss costs and premiums. Although in many property/casualty lines, insurers must be able to supply data according to statistical plans approved by the regulators, every insurer will maintain its data in its own way, and each insurer will have its own sources of data errors. One key element of the work of the ratemaking actuary is to enhance the data base. The actuary must be sure that information is recorded quickly and accurately, that data definitions are clearly understood and used appropriately, and that systems are appropriately designed to gather, store, and present the data in an efficient and accurate fashion. Data on claims, loss payments, and premiums may be collected and tabulated in any of the following three formats. (A fourth basis, called the ***report year***, which is used for claims-made policies will not be discussed here.) Throughout this chapter, we will assume all policies are written for a twelve-month period unless stated otherwise.

4.3.1 Accident Year

This is the most common method for compiling actuarial data. Any action on any claim arising from a loss event (accident) which occurred in calendar year Z will be accounted for as ***accident year Z***, regardless of the year in which the action was taken. The first annual report of total claims and losses incurred for accident year Z is compiled as of December 31, Z, and consists of the losses paid-to-date plus any unpaid loss reserves for accidents that occurred in calendar year Z. The first annual report is most dependent upon the reserve estimate. As time passes, all losses are paid and reserves eventually decline to zero. It is only at this ultimate stage that the actual claims and loss payments are fully known. Prior to that, the incurred data are "immature" and depend on reserve estimates. Some claims that occurred during accident year Z, may first be reported to the insurer many years after December 31, Z. Immature accident year data are available very soon after the end of the year.

4.3.2 Policy Year

Any claim action arising from a policy that became effective in calendar year Z will be accounted for as ***policy year Z***. At December 31, Z, policy year Z is not complete because typically most policies written in calendar year Z will not expire until after the end of that year.

Policy year Z is the accounting basis for all actions on all policies which took effect in calendar year Z, some as early as January 1, Z, and some as late as December 31, Z. For a policy issued early on January 1, Z, the policy year exposure period ends late on December 31, Z. On the other hand, for a policy issued late on December 31, Z, the exposure period does not end until December 31, $Z+1$. Thus the exposure period for policy year Z is the full 24 months from January 1, Z, through December 31, $Z+1$. Further, if all issue dates and loss occurrences are uniformly distributed, then the midpoint of the loss occurrence exposure period is midnight of December 31, Z (or 12:01 a.m. January 1, $Z+1$). Under these assumptions, Figure 4.1 illustrates the pattern of loss occurrence accounted for as policy year Z.

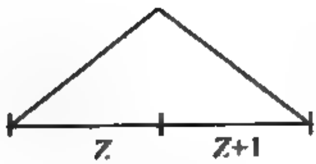


Figure 4.1

Hence, it is not possible to obtain complete policy year Z data until December 31, $Z+1$. Prior to that we will be dealing with an incomplete policy year and will be forced to estimate the complete policy year data. This is a significant disadvantage. As with accident year Z data, policy year Z data are comprised of losses paid-to-date plus estimated unpaid loss reserves, but in this case for all accidents that occurred on policies written during calendar year Z.

Policy year data have the advantage of allowing the pricing actuary to match on a consistent basis premiums and losses from one accounting basis (i.e., the policy year).

4.3.3 Calendar Year

Any accounting action that takes place in calendar year Z is accounted for as ***calendar year Z***, regardless of the year in which the corresponding policy was issued or the accident occurred. The results for calendar year Z can be finalized as of December 31, Z. Paid losses for calendar year Z are the total of all loss payments in that calendar year regardless of the date of occurrence of the accident or the issue date of the policy. Incurred losses for calendar year Z can be defined as

$$\begin{aligned}
&\text{Incurred Losses }(Z)\\
&\quad = \text{Paid Losses }(Z) + \Delta\text{ Reserves }(Z)\\
&\quad = \text{Paid Losses }(Z) + \text{Unpaid Loss Reserves } 12/31/Z\\
&\qquad\quad - \text{Unpaid Loss Reserves } 12/31/Z-1.
\end{aligned}$$

Thus, incurred losses for calendar year Z do not correspond to any particular accident period or any particular policy period. It is possible, however, to determine calendar year incurred losses immediately at the end of the

calendar year. Note that the Unpaid Loss Reserves include IBNR which is defined later in this chapter.

4.4 Premium Data

Premium data required for ratemaking are usually readily available and relatively accurate. The required data can be in either of two forms: written premiums or earned premiums.

Written premiums categorize premiums by the effective date of the policy. If, for example, the annual premium for a policy is \$120 and the policy anniversary is October 15, then \$120 would be recorded under written premiums for year Z (like cash accounting). The \$120 premium is not fully earned in year Z, however. In fact, if we ignore the effect of acquisition expenses, only \$25 of the \$120 is earned in year Z. The remaining \$95 is earned in year $Z+1$. Thus, ***earned premiums*** are the amounts actually earned in the period, which is analogous to accrual accounting. Data on calendar year earned and written premiums are almost always available. The difference between the written premium and the earned premium is the ***unearned premium***.

4.5 The Exposure Unit

Generally, the ratemaking actuary produces a rate manual that contains rates per ***unit of exposure***. The rate manual for auto insurance, for example, will list rates per car-year. For homeowners insurance (dwelling coverage), rates are often listed per 1000 of the value of the dwelling. In workers compensation insurance, the exposure unit for most classes is per 100 of payroll. The premium that is paid is the rate manual rate multiplied by the number of units of exposure. The exposure unit is defined in terms of an exposure base. In the above examples, the exposure bases are car-years, value of dwelling, and payroll, respectively.

A good exposure base should

- be an accurate measure of the quantitative exposure to loss,
- be easy for the insurer to determine (at the time the premium is calculated),
- not be subject to manipulation by the insured,
- be easy to record and administer, and
- be understood by the policyholder.

As is often the case in property/casualty work, the exposure base is a compromise among these criteria. For automobile insurance, for example, a better exposure base for criterion one would be annual mileage; however, it does not satisfy criterion two, three or four.

Another criterion that is advantageous for an exposure base, especially in times of rapid inflation, is that it should automatically adjust to changes in the cost of settling claims. In providing liability insurance for sporting events at a large stadium, for example, it may be preferable to set rates per dollar of gate receipts rather than per spectator. Receipts are usually easier to measure and will automatically adjust to inflation, allowing a set of rates to be effective for a longer time. The exposure base normally used for homeowners and workers compensation insurance will shift with inflation. The normal basis for automobile insurance (i.e., a car-year) will not.

Once an exposure base has been chosen, the actuary develops a rate per unit of exposure. The premium is then the product of the rate per unit and the number of units of exposure.

4.6 The Expected Effective Period

Operating a property/casualty insurance company can be compared to driving a car. The company president steers the car, the marketing vice president has the accelerator pressed to the floor, and the ratemaking actuary yells out directions while looking out the back window.

In reality, the ratemaking actuary must set rates that will be in effect for a future period, based on past information. The actuary will have data from an historic experience period. There will be time lags between the end of the experience period and the rate calculation date, and from the date of calculation to the effective date of the new rates, as illustrated in Figure 4.2.

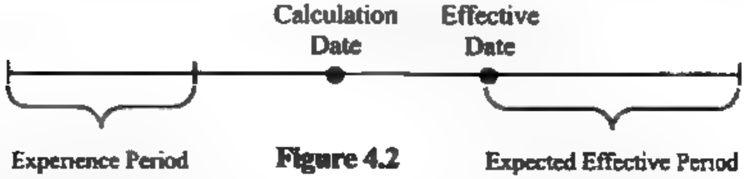


Figure 4.2

The ***expected effective period*** (e.g., the period during which losses may occur under policies to be written using the new rates, also known as the ***forecast period***) will be a policy period. If rates for annual policies, for example, are

changed once a year on the same date, then the expected effective period will be one policy year which (as explained in Section 4.3.2) will cover two calendar years in terms of the exposure to the occurrence of losses. As an illustration, if rates take effect on October 15, Z, and are to be effective for one year, and if the insurer only writes one-year policies, then the expected effective period may appear to be the year from October 15, Z to October 15, $Z+1$. Policies will, however, be in effect from October 15, Z to October 15, $Z+2$ (the latter being the expiration date of a policy issued October 15, $Z+1$). Loss occurrences could arise anywhere during that 24-month period. Thus, the mid-point of the expected effective period would be October 15, $Z+1$.

If the actuary is pricing for a policy year, the existence of policy year data would allow for an analysis that compares claims with the premiums that were collected to pay for those claims. Data from policy year Z are not complete (even for the first report) until December 31, $Z+1$, so either the actuary will have to work with at least one incomplete policy year or will be forced to work with relatively out-of-date data.

It is more common to work with accident year data, which then requires a different calculation of the trend factors (see Section 4.7.2) and loss-development factors (see Section 4.7.1). The use of accident year data means we are using more recent data than if policy year data were to be used. Further, claims data are almost always available in an accident year format.

Generally, calendar year data are not used in ratemaking. The payment of some claim checks in calendar year Z tells us little about how future policies should be priced. Thus the actuary normally chooses between policy year and accident year data, each with its own advantages and disadvantages.

4.7 INGREDIENTS OF RATEMAKING

4.7.1 LOSS-DEVELOPMENT FACTORS

The actuary, pursuing the rate making objective of responsiveness, will use the latest available data. Unfortunately, much of this data will be incomplete or immature in the sense that the final claim cost will not be known and a claim cost estimate must be used. In this regard, the actuary should check the

historical data to determine how accurate such past estimates have been. Consider, for example, the incurred loss data (paid plus reserves) shown in Table 4.1 for reported claims.

The numbers that were reported as of December 31, AY5 lie along the last diagonal of the data triangle. We can see that for this line of business, early incurred estimates were less than later estimates. There could be two reasons for this, either that (a) reserve estimates were deficient or optimistic, or (b) there were claims that were ***incurred but not reported*** (IBNR) in early report years that were subsequently reported and included in later report years. The phenomenon portrayed in Table 4.1 is called ***loss development***. The data can then be used to calculate the following incurred age-to-age loss-development factors.

Table 4.1

Incurred Losses for Reported Claims by Development Year					
Accident	Development Year				
Year	0	1	2	3	4
AY1	8,525	10,285	11,304	11,884	11,922
AY2	10,063	12,405	13,685	14,138	
AY3	12,265	14,101	15,633		
AY4	16,943	21,586			
AY5	20,175				

Table 4.2

Age-to-Age Incurred Loss-Development Factors Based on Reported Claims				
Accident Year	1/0	2/1	3/2	4/3
AY1	1.206	1.099	1.051	1.003
AY2	1.233	1.103*	1.033	
AY3	1.150	1.109		
AY4	1.274			
Average	1.216	1.104	1.042	1.003

* For example, $\frac{13,685}{12,405} = 1.103$

This technique is the same as the technique that was demonstrated for loss reserving in Chapter Three. In fact, the ratemaking actuary is well advised to seek the assistance of the reserving actuary in calculating and selecting loss-development factors.

Let us assume there is no further development beyond the fourth report, so that factors for later report durations would be 1.000. If we model the loss development by the arithmetic average of the factors presented in Table 4.2, then the estimates in Table 4.3 should be used for the expected ultimate dollars of loss to be paid by the time all claims are fully paid.

In setting rates, it is essential that the ultimate loss development be included. The premium charged for any cohort of policyholders must be enough to pay in full all of the losses that arise.

For at least two reasons it is possible to have incurred loss-development factors less than 1.000. First, if an insurer is very conservative in setting its case reserves, then actual ultimate losses will be less than the reserves would indicate. Second, in lines such as auto collision, homeowners, and fire, there will be recoveries after the insurer's initial loss payments because of salvage and subrogation.

The pricing actuary will also adjust the data so as to avoid too large an impact from one or two large losses (e.g., in one Territory). Losses may be capped at a specified level (e.g., $500,000) to avoid inappropriate distortions. Alternatively, large losses may be averaged over a longer period of time (e.g., five loss years). Again the purpose is to avoid inappropriate distortions from one or two large losses.

Table 4.3

Expected Ultimate Incurred Losses by Accident Year		
Accident Year		
AY1	11,922	
AY2	14,180	(14,138×1.003)
AY3	16,338	(15,633×1.042×1.003)
AY4	24,906	(21,586×1.104×1.042×1.003)
AY5	28,306	(20,175×1.216×1.104×1.042×1.003)

4.7.2 Trend Factors

The ratemaking actuary will need to estimate the ***expected loss cost*** for the future policy effective period to which the rates will apply. The estimate will be based on past experience period data, as illustrated in Figure 4.2. Therefore, past experience data must be adjusted to a cost level applicable to the future effective period. Reasons for cost changes between the past and future periods include not only economic inflation, but also changes in judicial decisions, changes in mandated benefits, technical advances, and so on. The actuary typically uses a trend factor to adjust past experience to future levels.

The data available to the actuary will normally include the illustrative data shown in Table 4.4, for a particular state or province.

Table 4.4

Ratemaking Data			
Accident Year	Earned Exposure Units	Ultimate Losses (Fully Developed)	Number of Incurred Claims*
AY1	1,085,644	129,620,410	55,810
AY2	1,096,235	146,865,366	58,706
AY3	1,126,283	146,290,566	59,822
AY4	1,144,318	181,457,324	64,636
AY5	1,205,142	227,430,574	69,474

* Note that the number of incurred claims used for ratemaking would need to be adjusted to ultimate values in the same way that incurred losses are adjusted to ultimate losses.

From these data, the values shown in Table 4.5 can be calculated by use of formulas (3.1a), (3.1b), and (3.2) from Chapter Three.

Table 4.5

Average Frequency, Average Severity, and Loss Cost			
Accident Year	Average Claim Frequency	Average Loss Severity	Loss Cost per Unit Exposure
AY1	.05141	2,323	119.39
AY2	.05355	2,502	133.97
AY3	.05311	2,445	129.89
AY4	.05648	2,807	158.57
AY5	.05765	3,274	188.72

Assume that the purpose of the analysis is to determine rates to take effect September 1, AY6 for one-year policies, and that rates will be in effect for one year. This means that the actuary is being asked to set a price for a future effective period whose midpoint (i.e., the average accident date) is September 1, AY7. How can we adjust the past experience data so as to estimate the expected loss cost (pure premium) as of September 1, AY7?

There are several potential answers to this question. It is the actuary's responsibility to choose the most appropriate methodology for the particular situation.

One mathematical tool that is frequently used is ***least squares***. The actuary will use least squares to fit a line or curve to the experience period data. Then, based on that curve, the actuary would determine trend factors to use for the loss cost estimate for the average accident date of September 1, AY7.

Having chosen a least-squares approach, there are still a variety of options for determining the trend factor. Some actuaries, for example, fit the curve to loss cost data that are a product of frequency and severity. Other actuaries fit separate curves to the frequency and severity data. The fit may be straight line, log-linear (which implies exponential growth), and so on. Exponential growth, which is analogous to compound interest, provides for a constant percentage growth model. Linear growth models result in decreasing percentage growth over time.

Beyond the technicalities of the curve fitting, the actuary could use ***regression analysis*** to predict a loss cost value on the curve for the average accident date of the future effective period. Depending on the method used, this predicted value could be estimated using a procedure that implicitly assigns equal weight to each year in the experience period. Actuaries normally prefer to give extra weight to the most recent data, so that approach could be modified. The following example illustrates two popular methods for projecting the historic experience period data to the future effective period.

EXAMPLE 4.1

Given the loss costs shown in Table 4.5, estimate the expected loss cost for rates that take effect September 1, AY6 on one-year policies. Assume that rates will be in effect for one year.

Solution

Accident Year	X = Year − $AY1$	Loss Cost [1]	ln(Loss Cost) [2]
AY1	0	119.39	4.782
AY2	1	133.97	4.898
AY3	2	129.89	4.867
AY4	3	158.57	5.066
AY5	4	188.72	5.240

Fitting a straight line to the ln(Loss Cost) data, we get

$$Y = 4.7534 + 0.1085X. \tag{4.1a}$$

The slope is .1085, which implies an exponential growth factor of 10.85% per year (i.e., $\delta = 0.1085$). Our projected loss cost could now be found as

$$\text{Projected Loss Cost} = (\text{Experience Loss Cost}) \cdot e^{\delta t},$$

$$\text{where } \delta = 0.1085. \tag{4.1b}$$

The determination of the time factor, t, still remains. The given experience period data (i.e., the loss costs) are accident year data, and the average accident date of accident year Z is June 30, Z. If the experience period data had been policy year data, then for policies issued January 1, Z to December 31, Z, the average accident date (of policy year Z) would be December 31, Z.

We are projecting to the average accident date of the expected effective period, which in this case is September 1, AY7. Given accident year data and using the straight line given by formula (4.1a), the loss cost corresponding to September 1, AY7 can be taken from that line using $X = 6.1\dot{6}$, and is 226.44.

---

[1] Fitting a straight line to Loss Cost would be the same as a linear extrapolation.

[2] Fitting a straight line to ln(Loss Cost) results in an exponential model which is normally preferable in modeling inflation.

Alternatively, we could use the slope factor of 0.1085 and the exponential curve given by formula (4.1b), and base our projected loss cost on only AY4 and AY5 loss costs as follows:

AY4 Loss Cost Projected to September 1, AY7 ($t = 3.1\dot{6}$):

$$(158.57) \cdot e^{0.1085(3.16)} = 223.58$$

AY5 Loss Cost Projected to September 1, AY7 ($t = 2.1\dot{6}$):

$$(188.72) \cdot e^{0.1085(2.16)} = 238.73$$

Projected Loss Costs Weighted 30/70 (for example):

$$(0.30)(223.58) + (0.70)(238.73) = 234.19$$ ❒

The above examples represent only two of a large number of possible defensible results. There is no uniquely correct answer or approach. Further, the trend factor should not be determined purely from a "black box" mathematical calculation. As stated earlier, the professional actuary must take into account a variety of external forces that could affect future lost costs. Examples include:

- recent court adjudications (e.g., a change in the legal no-fault threshold),
- interpretation of recent legislative changes (e.g., a change in speed limits)
- an expected change in the rate of economic inflation,
- the level of economic activity (e.g., during a recession workers compensation and fire claims tend to rise, whereas automobile claim rates fall),
- recent changes in underwriting criteria or definitions, and
- changes in mandated benefit levels.

Management and regulators may respond negatively if the trend factor is determined in a purely subjective fashion. The inclusion of nonmathematical factors is imperative, however, and will prove to be defensible.

The trend factor is normally the single most important input parameter in the ratemaking process and one of the few that allows for significant subjective modification. The actuary is justified in spending a considerable amount of

time and effort on this aspect of ratemaking, and will usually invite input from other actuaries and other members of the management team.

Some managers and regulators believe that inflation is double-counted when both trend and loss-development factors are applied. Figure 4.3 can be used to dispel this "overlap fallacy."

Figure 4.3

The trend factor is used to effectively adjust the experience period indications from the average accident date of the past experience period to the average accident date of the future effective period. The loss-development factor is used to adjust immature losses from their latest reported values to their expected ultimate values. Hence, there is no overlap in the two factors; each is essential and stands alone.

Some actuaries will also determine trends in premiums to use in the Loss Ratio method. There can exist premium trends if, for example, there is a consistent shift in the policies that people choose toward higher deductibles (a premium decrease) or higher limits of liability (a premium increase) or simply if policyholders replace older cars with newer, more expensive cars (a premium increase). Further discussion of premium trending is beyond the scope of this textbook.

4.7.3 Expenses

The rate being set by the actuary must also include an allowance for expenses. Expenses are usually divided into at least two categories and allocated accordingly. The first category includes expenses associated with the loss adjustment and payment process (e.g., lawyers' fees, claims adjusters' fees, and court costs). They are called ***loss adjustment expenses*** (LAE), and are accounted for as part of incurred losses. Loss adjustment expenses associated with a particular claim (e.g., lawyers' fees) are so allocated and are called ***allocated loss adjustment expenses***. These allocated loss adjustment expenses (ALAE) typically exhibit behavior similar to losses

(i.e., as to amount and timing) and are usually combined with losses for ratemaking. Loss adjustment expenses (e.g., the salary of the head office claims department manager) that are not associated with a particular claim are called ***unallocated loss adjustment expenses*** (ULAE). They are included in the incurred loss data using a formula approach beyond the scope of this text. Suffice it to say that incurred losses should include all loss adjustment expenses expected to be incurred up to the final loss payment date.

The criteria to determine whether an expense is allocated or unallocated are subject to some variation. Under Statutory Accounting in the United States, the current definition is based on the function rather than source of the expense. New terminology, in fact, has been created to emphasize the distinction. Expenses associated with providing claim defense and reduction of final settlement amounts (such as use of nurses as case managers to coordinate the various medical providers used by the claimant, and to monitor that care is given to minimize the time away from work) are now called Defense and Cost Containment (DCC) expenses, while expenses associated with claims adjustment are considered Adjusting and Other (AAO) expenses. Under this definition, attorneys employed by the insurer who provide legal defense are part of DCC. If these same attorneys are used as adjusters (investigation, setting case reserves, etc.) they would be considered part of AAO. In general, DCC are considered ALAE and AAO are considered ULAE. The most important point is that the definitions of the data used for ratemaking or for reserving should be consistent over time, or at least be capable of being adjusted to a consistent basis.

The second category of expenses to be accounted for includes commissions, premium taxes, licenses and fees, and head office expenses. Many of these, particularly commissions, premium taxes, and an allowance for profit and contingencies, are a defined percentage of the gross rate. Because of this, many insurers formulate *all* expenses, other than loss adjustment expenses, as a percentage of the gross rate. This leads to the following formulas for calculating the gross rate.

$$\text{Expense Ratio} = \begin{array}{l}\text{All Expenses Other Than LAE,}\\ \text{as a Percentage of the Gross Rate}\end{array} \tag{4.2a}$$

$$\text{Permissible Loss Ratio}^{3} = 1 - \text{Expense Ratio} \tag{4.2b}$$

[3] Some authors call this the target, balanced point, or expected loss ratio.

$$\text{Gross Rate} = \frac{\begin{array}{c}\text{Incurred Loss Cost per Unit of Exposure}\\ \text{(Trended and Developed)}\end{array}}{\text{Permissible Loss Ratio}} \qquad (4.2c)$$

If the expenses that are a percentage of the gross rate are removed, what is left is the amount of the gross rate expected to be needed to pay all losses and LAE through the final payment date.

Some insurers, at least for some lines, separate expenses into those that vary (V) directly with the gross rate (e.g., commissions), and those that are fixed (F) per unit of exposure (e.g., salaries and other overhead). In this case, we have

$$\text{Gross Rate} = \frac{\begin{array}{c}\text{Incurred Loss Cost per Unit of Exposure}\\ \text{(Trended and Developed)}\end{array} + F}{1 - V} \qquad (4.3)$$

Exercise 4.11 illustrates a variation to formula 4.3. It also illustrates how pricing can be affected by legislation. Read the question carefully.

The examples that follow assume that all expenses (other than LAE) are a percentage of the gross rate, unless indicated otherwise.

4.7.4 Loading for Profit and Contingencies

The incurred loss cost in the gross rate formula (4.2c or 4.3) is the expected value of future losses. Associated with this expected value is a variance of future losses. Because the loss cost is a random variable, the actual losses are likely to be different from the expected losses.

It is normal, therefore, to include a provision for adverse deviations in the rate. There is also generally a specific profit provision in the rate. In total, these are referred to as the ***loading for profit and contingencies.***

This loading may be included in the calculation implicitly or explicitly. Under the ***implicit approach***, the actuary will choose conservative input parameters and methodologies throughout the calculation. The expectation is then, on average, that actual experience will be better than expected experience, resulting in a profit. Under the ***explicit approach***, the actuary will calculate the best estimate of the loss cost with no level of conservatism, but then add an explicit factor (usually a percentage of the gross rate) for profit and contingencies.

The explicit approach to providing for profit and contingencies is becoming more popular, and is the normal approach for rates that are filed with regulatory authorities. Until recently, a common approach was to use best estimates for all variables except investment income, which was ignored in setting rates. The investment return then became the loading for profit and contingencies. Now, however, many regulators require the expected total return be used during the rate approval process.

Because of the competitive nature of the property/casualty business, the marketplace should not allow excessive profit and contingency margins. On one hand, if an insurer's rates are too high, it will lose business, especially the very best risks. On the other hand, if rates are too low, the insurer can expect to have underwriting losses (operating results before investment gains) that could ultimately force it out of business, if not corrected.

4.7.5 Credibility Factors

The actuary is often forced to deal with sparse data. In the first section of this chapter, two somewhat contradictory objectives of ratemaking—responsiveness and stability—were noted. If the actuary wishes to emphasize responsiveness, the most recent data should be used. If, however, these data are sparse, the actuary may prefer a greater volume, which may require using older data. As we will now see, however, there are other, more imaginative, solutions available.

One tool that aids the actuary in striking a compromise between responsiveness and stability when working with sparse data is the ***credibility factor***. In Chapter One we noted that the law of large numbers states that as the number of observations increases, the difference between the observed frequency of an event and the true underlying frequency tends to zero. Slightly restated we could say that the more data we have, the more credible are the indications (i.e., the more we would tend to believe what the data indicate). The credibility of data is a function of the uncertainty associated with the underlying frequency and severity distributions. If, for example, losses range from 0 to 25,000, we might say that at least 1000 claims would give a fully credible indication of the distribution. On the other hand, if losses range from 0 to 1,000,000, we might require at least 3,000 claims for full credibility.

This discussion leads to the credibility factor Z, which has the following properties:

(1) $0 \le Z \le 1$. If $Z = 1$, the data are considered to be fully credible, and if $Z = 0$, the data are considered to have no credibility for ratemaking purposes.

(2) $\frac{dZ}{dE} > 0$, which implies that credibility increases as the volume of experience data increases.

(3) $\frac{dZ}{dE}\left(\frac{Z}{E}\right) < 0$, which implies that the percentage change in credibility, due to a policy or claim of a given size, should decrease as the size of the risk increases. The addition of 100 units of exposure, for example, where exposure is now 500 units, will have more effect than the addition of 100 units where exposure is now 10,000 units.

Two common credibility formulae are

$$Z = \frac{E}{E+K}, \quad k > 0 \quad \text{so that } 0 \le z \le 1 \tag{4.4a}$$

and

$$Z = \min\left(\sqrt{\frac{n}{k}}, 1\right) \tag{4.4b}$$

where

- E is some measure of ***exposure*** (or ***evidence***) such as premium income,
- K is a constant which varies according to the inherent variability of the line of business,
- n is some measure of exposure such as number of claims, and
- k is the number of claims required for full (i.e., 100%) credibility for a given line of business.

In Canadian auto ratemaking, for example, common credibility formulae are

$$Z = \sqrt{\frac{n}{3{,}246}}, \quad 0 \le Z \le 1, \tag{4.5a}$$

for third party liability, and

$$Z = \sqrt{\frac{n}{1{,}082}}, \quad 0 < Z \le 1, \tag{4.5b}$$

for collision. The higher value of k for third party liability is appropriate since there is more variability in liability than in collision losses.

Note that in using formula (4.5a), with $0 \leq Z \leq 1$, a statistical indication based on 3,246 claims would be assigned full credibility (i.e., $Z = 1$). A statistical indication based on only 812 claims would be assigned 50% credibility, and so on. Further analysis of credibility factors is beyond the scope of this text.

The application of credibility factors will be illustrated later in this chapter.

4.7.6 Investment Income

A basic principle of actuarial science is that one must explicitly account for the time value of money. The premium to be charged for an expected $1,000,000 liability loss should be significantly smaller than the premium for an expected $1,000,000 fire loss, if the fire claim is paid within a few months but the liability claim takes several years to settle.

The process of identifying the timing of loss payments and a methodology for discounting such cash flows was more appropriately addressed in Chapter Three, Loss Reserving.

4.8 Rate Changes

There are three steps to a rate change. First, we determine the average, or overall, rate change required. Second, we decide on changes required in the differentials that apply to the rate classification parameters (e.g., class or territory). Finally, having made these two independent calculations, we adjust the results so that the overall change in premium income is actually the one needed. These three steps are discussed in detail in the following three subsections.

4.8.1 Overall Average Rate Change

Two methods can be used to calculate the overall average rate change: the ***loss cost*** (or ***pure premium***) method and the ***loss ratio method***.

Loss Cost (or Pure Premium) Method

In this approach we first find

$$\text{New Average Loss Cost} = \frac{\text{Expected Dollar Losses in Effective Period (Trended and Developed)}}{\text{Number of Earned Exposure Units}}, \tag{4.6a}$$

which would lead to

$$\text{New Average Gross Rate} = \frac{\text{New Average Loss Cost} + \text{Fixed Expense Per Exposure}}{\text{Permissible Loss Ratio}}, \tag{4.6b}$$

where Permissible Loss Ratio $= 1 - \text{Expense Ratio}$. Fixed and variable expenses should to be separately identified for the ratemaking calculation. As a result, the Permissible Loss Ratio would be

$$\begin{aligned}\text{Permissible Loss Ratio} &= 1 - \text{Variable Expense Ratio} \\ &\quad - \text{Profit and Contingencies Ratio}.\end{aligned} \tag{4.6c}$$

Loss Ratio Method

In this approach we begin with

$$\text{Indicated Rate Change} = \frac{\text{Expected Effective Loss Ratio} + \text{Fixed Expense Ratio}}{\text{Permissible Loss Ratio}} - 1, \tag{4.7a}$$

where

$$\text{Expected Effective Loss Ratio} = \frac{\text{Expected Dollar Losses in Effective Period (Trended and Developed)}}{\text{Dollars of Earned Premium at Current Rates}}, \tag{4.7b}$$

$$\text{Fixed Expense Ratio} = \frac{\text{Fixed Expense Per Exposure}}{\left(\dfrac{\text{Dollars of Earned Premium at Current Rates}}{\text{Number of Earned Exposure Units}}\right)} \qquad (4.7c)$$

The denominator of formula (4.7b) requires some explanation. The indicated rate change is applied to today's existing rate manual, so the expected effective loss ratio must use earned premium at the current rates or rate level. The earned premium data from the company's accounting records could include some earned premium from previous rate manuals.

There are two approaches to calculating the earned premium at current rates. The first is to reproduce the rate manual by computer and calculate what the earned premium would have been, given the historical portfolio of business (i.e., the distribution of earned exposures). This method is called ***extending exposures*** and can be expressed as

$$\text{Dollars of Earned Premium at Current Rates} = \sum_{ijk} CR_{ijk} \cdot e_{ijk}, \qquad (4.8)$$

where

- CR_{ijk} is the current rate for cell ijk defined by rate classification parameters i, j, and k (e.g., class, territory, rate group), and
- e_{ijk} is the corresponding number of earned exposure units in cell ijk in the historical book of business.

The second approach to calculating earned premiums at current rates is to adjust the insurer's earned premium accounting entry to reflect the effect of all rate changes that were made subsequent to the time the earned premiums were written. This method is known as the ***parallelogram method***. It can be understood more easily by considering the diagrams in Example 4.2.

EXAMPLE 4.2

Consider the following data:

Calendar Year	Earned Premium
CY3	3,853
CY4	4,600
CY5	5,125

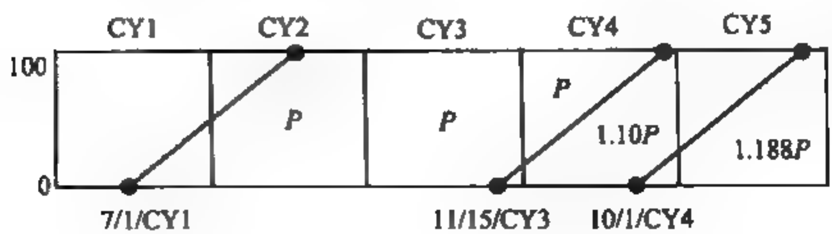
Assume all policies have a one-year term, the policy issues are uniformly distributed, and the following rate changes have occurred:

Date	Rate Change
July 1, CY1	+12.5%
November 15, CY3	+10.0
October 1, CY4	+ 8.0

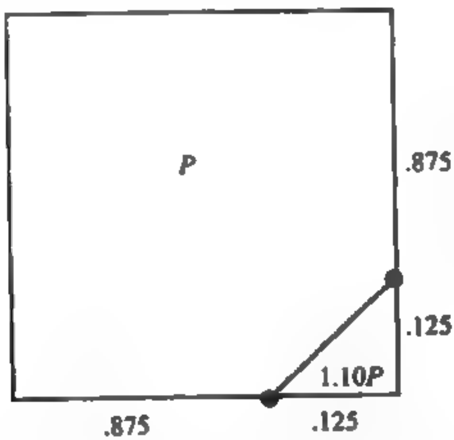
Rates are currently at the level set on October 1, CY4. Calculate the earned premium at current rates for calendar years CY3, CY4, and CY5.

Solution

To find the solution, we will use the parallelogram method illustrated below. Each vertical line denotes a calendar year division. Policy effective dates start on the bottom horizontal line and move linearly to expiration on the top horizontal line. Thus, the diagonal lines represent policy periods associated with rate changes. The vertical axis represents the percentage of the policy exposure earned at time t (from 0% to 100%).



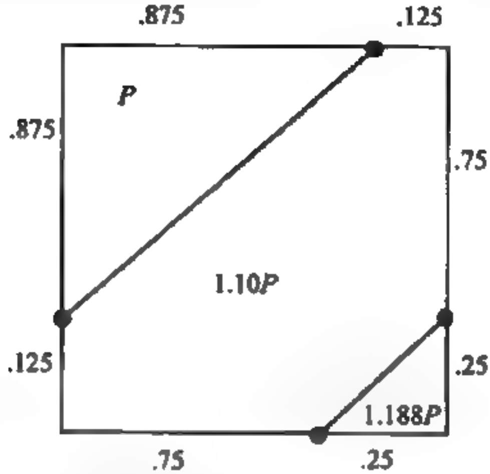
Let P denote the rate level effective July 1, CY1. By July 1, CY2, all earned premium would be on this new basis. Rates are increased by 10% on November 15, CY3, but it takes until November 15, CY4 before all earned premium is on at least this new basis. Call this $1.10P$. Similarly, the rate change of +8.0% on October 1, CY4 will not be fully reflected in earned premium until October 1, CY5. The current rate level will be $(1.10)(1.08)P = 1.188P$. We wish to bring all earned premium of CY3, CY4, and CY5 to the current level of $1.188P$. For premiums earned in calendar year CY3, two rate levels were in effect as seen in the following diagram.



The area of the lower right triangle of this unit square is $\frac{\text{Base} \times \text{Height}}{2} = 0.0078125$, and the remaining area within the square is 0.9921875.

That is, 0.78125% of the earned exposure was at rate level $1.10P$ and 99.21875% of the earned exposure was at rate level P. So the earned exposure in CY3 was at an average rate level of $(0.0078125)(1.10P) + (0.9921875)(P) = 1.00078125P$. But the current rate level is $1.188P$. Thus, CY3 earned premiums must be multiplied by 1.188/1.00078125 to bring them to the current rate level. This assumes the policies are issued uniformly throughout the year.

In CY4, premiums were earned under three rate levels as shown in the following diagram.



By calculating the areas of the triangles and the irregular interior region, we can determine weights of 0.3828125 for P, 0.5859375 for $1.10P$, and 0.03125 for $1.188P$, for calendar year CY4. This leads to an average earned premium in CY4 of:

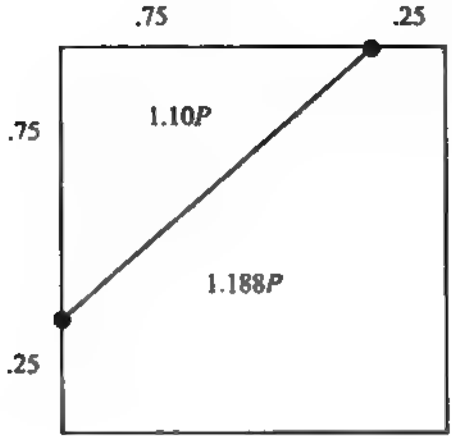
$$0.3828125P + (0.5859375)(1.10P) + 0.03125(1.188P) = 1.06446875P.$$

Since we want the current rate level of $1.188P$, the on-level factor for CY4 is $1.188/1.0644688$.

Finally, the CY5 average earned premium is

$$(0.28125)(1.10P) + (0.71875)(1.188P) = 1.16325P,$$

as developed from the following diagram.



So the CY5 on-level factor is 1.188/1.16325.

These on-level factors for each calendar year are then applied to their respective calendar year actual earned premium to generate approximate earned premium at current rates, as shown in the following table.

Year	Earned Premium	On-Level Factor	Earned Premium at Current Rates
CY3	3,853	1.188/1.00078125	4,574
CY4	4,600	1.188/1.06446875	5,134
CY5	5,125	1.188/1.16325000	5,234

This method assumes a uniform distribution of policy issue dates over any calendar year. To the extent that this is not true, the results will be inexact and adjustments to this method will need to be made. □

EXAMPLE 4.3

Calculate the new average gross rate given the following information.

Expected Effective Period Incurred Losses (Trended and Developed)	30,000,000
Earned Exposure Units	1,000,000
Earned Premium at Current Rates	45,000,000
Present Average Manual Rate	45
Fixed Expenses	5,000,000
Fixed Expense Per Exposure Unit	5

Permissible Loss Ratio
= (1 − Variable Expense Ratio − Profit and Contingencies Ratio)
= 0.75

Solution

Loss Cost Method

$$\text{Expected Effective Loss Cost} = \frac{30{,}000{,}000}{1{,}000{,}000} = 30$$

$$\text{Indicated Average Gross Rate} = \frac{30+5}{0.75} = 46.67$$

Loss Ratio Method

$$\text{Expected Effective Loss Ratio} = \frac{30{,}000{,}000}{45{,}000{,}000} = .66\dot{6}.$$

This shows that at current rates we would expect a loss ratio of $.66\dot{6}$. Comparing this to the permissible loss ratio, we find

$$\text{Indicated Rate Change Factor} = \frac{0.66\dot{6} + \frac{5}{45}}{0.75} - 1 = +3.7037\%,$$

which indicates a rate increase of 3.7037%. Thus we find

$$\text{Indicated Average Gross Rate} = 45(1.037037) = 46.67. \quad \square$$

Theorem

The loss ratio method and the loss cost method will lead to the same answer, given consistent and reconcilable data, and the use of the same permissible loss ratio. Assume we have data split according to three risk class variables such that there are i classes for the first risk class variable, j classes for the second variable, and k classes for the third variable.

Proof

Loss Cost Method

Indicated New Average Gross Rate

$$= \frac{(\text{Expected Effective Dollars of Losses})\big/\sum e_{ijk} + F}{PLR}$$

$$= \frac{\sum \ell_{ijk}\big/\sum e_{ijk} + F}{PLR},$$

where F is the dollars of fixed expenses per unit of exposure, PLR is the permissible loss ratio, ℓ_{ijk} is the expected effective dollars of loss for cell ijk and e_{ijk} are the earned exposure units in risk cell ijk.

Loss Ratio Method

Indicated Average Gross Rate

$$= R_C \left(\frac{\text{Expected Effective Loss Ratio} + F\big/R_C}{PLR} \right),$$

$$\text{Current Average Gross Rate} = R_C = \frac{\sum_{ijk} CR_{ijk} \cdot e_{ijk}}{\sum_{ijk} e_{ijk}},$$

And

Expected Effective Loss Ratio

$$= \frac{\text{Expected Effective Dollars of Losses}}{\text{Dollars of Earned Premium at Current Rates}} = \frac{\sum_{ijk} \ell_{ijk}}{\sum_{ijk} CR_{ijk} \cdot e_{ijk}}.$$

Thus we find

Indicated Average Gross Rate

$$= R_C\left(\frac{\sum \ell_{ijk}}{\sum_{ijk} CR_{ijk}\cdot e_{ijk}} + F\big/R_C\right)\left(\frac{1}{PLR}\right)$$

$$= \frac{\sum \ell_{ijk}\big/\sum e_{ijk} + F}{PLR}$$

= Indicated Average Gross Rate by the Loss Cost Method. □

In practice, the actuary will usually do two separate average rate calculations. One is based on data from the actuary's own company; the other is based on a wider data base, which is normally available from one or more rating bureaus and/or statistical agencies. Unless the actuary's company is very large, the indication based on its data will have credibility $Z<1$. Then the final answer will be

Credibility-Weighted Average Indicated Rate
$= Z$(Company Indication) + $(1-Z)$(Rating Bureau Indication). (4.9)

4.8.2 Changing Risk Classification Differentials

Section 4.8.1 outlined standard methods for determining the indicated overall average rate change. The ultimate goal, however, is to determine new rates for each rating class that measure its potential for loss and, in the aggregate, will generate the indicated overall average rate change.

In that regard, it is useful to first explain how a rate manual is normally produced. For some line of business, one risk classification cell is chosen as the ***base cell***. It normally has the largest amount of exposure, so it will have maximal statistical credibility. The rate for the base cell is referred to as the ***base rate***. Other rate cells will be defined by a variety of risk classification variables, such as class, territory, and so on. For each risk classification variable, there will be a vector of ***differentials*** (or ***relativities***), with the base cell characteristic always assigned a differential (relativity) of 1.000. Assume, for example, there are three risk classification variables, x, y, and z, with i differentials for variable x, j differentials for variable y, and k differentials for variable z. By setting one base rate and three vectors of differentials for characteristics x, y, and z, a total of i, j, k rates are produced.

In the examples that follow, it is assumed that the differentials are multiplicative.

Each time rate levels are changed, the actuary reviews the available data and determines the need for a change in existing differentials, using either a loss cost approach, a loss ratio approach, or possibly both. The principles of a differential change are illustrated in the following example.

EXAMPLE 4.4

Determine new differentials for Class B and Class C, given the following information and assuming full credibility in all classes.

Class	Existing Differential	Experience Period Loss Ratio at Current Rates	Experience Period Loss Cost
A	1.00	0.65	129
B	0.85	0.71	120
C	1.21	0.66	157

Solution

Loss Ratio Method

If the experience period loss ratio at current rates for all three classes had been the same, it would have indicated that the existing differentials were exactly correct. The magnitude of the ***experience period loss ratio*** (versus the permissible loss ratio) is irrelevant at this stage. That relationship indicated the need for an overall average rate change. At this stage the goal is only to determine differentials among the risk classes. Class A is the base class and the Class A differential will remain at 1.00. The loss ratios in Class B and Class C, relative to Class A's loss ratio, indicate a need to increase the Class B and Class C differentials using

$$\text{(Indicated Differential)}_i = \text{(Existing Differential)}_i \times \frac{LR_i}{LR_{Base}}. \qquad (4.10)$$

Thus we find

$$(\text{Indicated Differential})_B = (0.85)\left(\frac{0.71}{0.65}\right) = 0.93$$

for Class B, and

$$(\text{Indicated Differential})_C = (1.21)\left(\frac{0.66}{0.65}\right) = 1.23$$

for Class C.

Loss Cost Method

Whereas the loss ratio method indicates how much to change old differentials, the loss cost method indicates the new differentials directly. The formula underlying the loss cost method is

$$(\text{Indicated Differential})_i = \frac{(\text{Loss Cost})_i}{(\text{Loss Cost})_{Base}}. \qquad (4.11)$$

Thus we find

$$(\text{Indicated Differential})_B = \frac{120}{129} = 0.93$$

for Class B, and

$$(\text{Indicated Differential})_C = \frac{157}{129} = 1.22$$

for Class C. □

In Example 4.4, the differentials calculated by the two methods were not exactly the same. This can happen because there are several risk classification characteristics $x, y, z \ldots$. If the distribution of all ***cross-variables*** in any cell is homogeneous, then the indications provided by the two methods will be the same (see the Appendix). If, however, the cell population is heterogeneous, then the answers without appropriate adjustment will differ. This will be illustrated in Examples 4.6 and 4.7.

Assume, for example, that young males, sports cars, and port cities all have high risk attributes and result in three large differentials (and correspondingly large rates). Assume also that a disproportionate number of young males drive sports cars, or that a disproportionate number of young males live in port cities. In this case, the loss cost method, applied independently for each

risk classification variable, would lead to large differentials for each of young males, sports cars, and port cities, and the premiums would then double-count the effect of the heterogeneous cross distribution of variables and overly penalize young male drivers of sports cars in port cities.

The loss ratio method, however, is somewhat self-adjusting, since the earned premium denominator of the experience period loss ratio is already larger to reflect the disproportionate number of young male drivers of sports cars in port cities. Other methods exist that mathematically minimize this potential bias in the loss cost approach, but they are beyond the scope of this text.

Example 4.5 illustrates the problem of heterogeneous cross distribution in some detail.

EXAMPLE 4.5

You are the consulting actuary for a property/casualty insurer that proposes to begin selling automobile insurance in State Z, which has three classification territories. The insurer wants advice on what territorial differentials to adopt. You have the following information showing the present average territorial differentials used by the top five insurers.

Territory	Differential
1	1.00
2	0.95
3	1.25

The following industry statistics are also available for the latest calendar year.

Territory	Cars Insured	Earned Premium (in thousands)	Incurred Losses (in thousands)
1	65,354	12,046	7,215
2	56,182	10,093	5,987
3	24,858	5,840	3,580

What territorial differentials would you recommend, and what comments would you include in your report?

Solution

Given the available data, approximations to standard loss ratio and loss cost approaches to establishing new differentials are possible.

Loss Ratio Method

Territory	Earned Loss Ratio	Ratio to Territory 1 Loss Ratio	Existing Differential	Indicated Differential
1	0.599	1.000	1.00	1.00
2	0.593	0.990	0.95	0.94
3	0.613	1.023	1.25	1.28

There is a problem with what we have done here. The calculation of the loss ratio uses industry earned premium in the denominator, whereas the correct calculation would use the insurer's earned premium at current rates. If every insurer had used territorial differentials identical to the given average of the top five insurers for a period long enough to make the loss ratio based on earned premium equal to the loss ratio based on current premium, then this calculation would have been correct. To the extent that this assumption is incorrect, then so too are the results. To check the accuracy of this assumption, we compare the average earned premium per car insured in the three territories.

Territory	Average Earned Premium Per Car Insured	Ratio to Territory 1
1	184.32	1.00
2	179.65	0.97
3	234.93	1.27

We would have more faith in the assumption if the ratios of these average earned premium figures were closer to 1.00, .95, and 1.25. This, however, may be the best that we can do, and the client should be aware of these limitations.

Loss Cost Method

Territory	Loss Cost per Car Insured	Ratio to Territory 1
1	110.40	1.00
2	106.56	0.97
3	144.02	1.30

The loss cost method, given the information we have, suffers from the possibility of heterogeneity in the cross variables. It might be possible, for example, that all young male owners of sports cars in State Z live in Territory 3. We have no way of knowing.

The evidence of the data suggests that the Territory 2 differential might stay at 0.95, and the Territory 3 differential might move upward toward 1.30.

As pointed out at the end of Section 4.8.1, when data are not fully credible we usually adopt a credibility-weighted result. For differentials, formula (4.9) is modified to

$$\begin{aligned}&\text{New Differential}\\&\quad = Z(\text{Indicated Differential})\\&\qquad\qquad + (1-Z)(\text{Existing Differential}).\end{aligned} \tag{4.12}$$

This provides another reason for moving to a differential of less than 1.30 for Territory 3 and keeping the Territory 2 differential of 0.95. □

To summarize what has been done to this point, visualize the baking of a birthday cake to be served to many invitees. The first requirement is to determine how big the cake should be, or how much batter is needed. This is analogous to determining the overall average rate change.

Next is the desire to provide each invitee with a perfect size piece of the cake. The cake can be sliced in many ways, but let us assume there will be three cuts: length, width, and height. These cuts parallel the risk classification variables x, y, and z. Some cuts are very close to other cuts and some are farther away, but all are straight lines. There are i length cuts, j width cuts, and height k cuts.

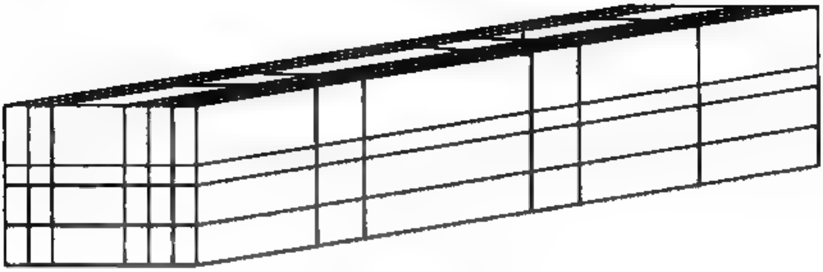


Figure 4.4

The end result is $i \cdot j \cdot k$ pieces of cake. The individuals with characteristics x_i, y_j, and z_k get their own personalized piece of cake of volume $l_i \cdot w_j \cdot h_k$. *On average*, everyone gets the needed piece of cake. Unfortunately, because we cut the cake with straight lines, not everyone gets exactly the size of piece he or she deserves. But this method comes very close, close enough for consumer satisfaction in a normal free market.

4.8.3 Balancing Back

The approach to ratemaking as outlined thus far has two phases. First, the actuary determines the need for an overall average rate change. Then, as outlined, the actuary independently assesses the need for a change in differentials while holding the base cell differential equal to 1.00.

This two-stage process can result in ***off-balance***. Assume, for example, the indicated overall rate change is +10%. Then assume that the actuary independently uses the data of Example 4.4 and the loss ratio method to arrive at the new differentials shown in the following table.

Table 4.6

Class Differentials			
Class	Existing Differential	Proposed Differential	Earned Exposure Units
A	1.00	1.00	410
B	0.85	0.93	395
C	1.21	1.23	195
			1000

The distribution of business is also given in Table 4.6. As expected, the base cell has the largest exposure.

Assume the target overall average rate change is +10% and the existing Class A premium is 100. Then the earned premium at current rates would be

$$100[(1.00)(410) + (0.85)(395) + (1.21)(195)] = 98{,}170.$$

If the base rate changes by +10%, then with the new differentials the earned premium would be

$$110[(1.00)(410) + (0.93)(395) + (1.23)(195)] = 111{,}892,$$

which represents a 13.98% increase in earned premium.

This apparent error results from the off-balance created when we change differentials (both upward) independently of the overall average rate change, and do not then ***balance back***. The resulting ***off-balance factor*** is the ratio $\frac{\text{New Average Differential}}{\text{Old Average Differential}}$, and the ***balance-back factor*** is its reciprocal. The balance-back factor is multiplied by the overall average rate change factor to determine the rate change factor to apply to the current base class rate. When the new differentials for the non-base classes are multiplied by the adjusted base class rate, the overall average rate change that was originally indicated will be realized.

In the above illustration, the change in differentials leads to an off-balance factor of

$$\frac{\text{New Average Differential}}{\text{Old Average Differential}} = \frac{(1.00)(0.410) + (0.93)(0.395) + (1.23)(0.195)}{(1.00)(0.410) + (0.85)(0.395) + (1.21)(0.195)} = 1.0361618.$$

With no change in the base rate, the change in differentials would increase premium income by an extra 3.6%. Assuming a 10% average rate change and no balance back, premium income would increase by (1.10)(1.0361618) = 1.1398, or +13.98%. Hence, to achieve the target 10% rate increase, the actuary should increase the base rate not by 10%, but rather by the factor

$\frac{1.10}{1.0361618} = 1.061603$, a 6.16% increase in the base rate. This would lead to the results shown in Table 4.7.

Table 4.7

Proposed Rates after Balance Back			
Class	Existing Rate	Proposed Rate	Earned Exposure Units
A	100	106.16	410
B	85	98.73	395
C	121	130.58	195

The earned premium at current rates is 98,170. The earned premium at the new rate and differential levels would be

$$(106.16)(410) + (98.73)(395) + (130.58)(195) = 107{,}987,$$

for an effective rate level increase of

$$\frac{107{,}987}{98{,}170} - 1 = 0.10,$$

the desired 10% increase.

It will be shown in the following examples that under the loss cost method the balance back can be achieved by adding one step, namely,

$$\text{Base Gross Rate} = \frac{\text{Average Gross Rate}}{\text{Average Differential}}. \tag{4.13}$$

4.8.4 Summary Of The Rate Change Procedure

In summary, a rate change is a three-step process in which we

- determine the overall average rate change,
- change all differentials as indicated, and
- balance back to the overall rate change indicated.

Normally, actuaries do not use their own company data to determine risk classification differentials, but rather they use industry-wide data if available. Even with industry-wide data, some cells will lack full statistical credibility. In that case, the actuary may only adopt a change in a differential to the extent

that the indicated change is statistically credible. Consistent with Example 4.5, this leads to formula (4.12).

EXAMPLE 4.6

Given the following information, calculate the proposed Class 1A rate for Territory 2. Class differentials will not be changed, and the province wide rate change is 5%. The base cell is Territory 1, Class 1A.

Territory	Exposure Earned Units	Average Existing Rate	Loss Cost	Average of Existing Class Differentials within Territory
1	2000	250	200	1.50
2	1000	500	300	1.25

Solution

Loss Ratio Method

Territory	Earned Exposure Units	Existing Average Rate	Loss Cost	Average of Existing Class Differentials within Territory
1	$\frac{250}{1.50}=166.67$	1.000	$\frac{400,000}{500,000}=0.8$	1.000
2	$\frac{500}{1.25}=400.00$	2.400	$\frac{300,000}{500,000}=0.6$	$2.4\times\frac{0.6}{0.8}=1.800$

Loss Cost Method

We must account for the fact that the class distribution is not the same in the two territories. This can be done by adjusting the loss cost for the average class differential in each territory.

Territory	Base Rate	Existing Differential	Loss Ratio	Indicated Differential
1	$\frac{250}{1.50}=166.67$	1.000	$\frac{200}{1.50}=133.33$	1.000
2	$\frac{500}{1.25}=400.00$	2.400	$\frac{300}{1.25}=240.00$	$\frac{240}{133}=1.800$

Note: 166.67 is the base rate for Territory 1, Class 1A.

$$\text{Balance-Back Factor} = \frac{\text{Old Average Differential}}{\text{New Average Differential}}$$
$$= \frac{(2000)(1.50) + (1000)(2.40)(1.25)}{(2000)(1.50) + (1000)(1.80)(1.25)}$$
$$= 1.1428571$$

New Territory 1, Class 1A base rate:
(166.67)(1.05)(1.1428571) = 200

New Territory 2, Class 1A rate: (200)(1.80) = 360 □

EXAMPLE 4.7

Given the following information, and assuming the revised rates take effect July 1, CY7 for one year on one-year policies, determine new rates for each of Class 1 and Class 2, for each of Territory 1 and Territory 2. (Class 1/2 differentials will not change.) Use the loss ratio and loss cost methods, and base the overall average rate change on CY5 policy year data, assuming they are fully credible for that purpose. The permissible loss ratio is 0.60.

Policy Year CY4 Losses

As of December 31, CY5		As of December 31, CY6	
Paid	Outstanding	Paid	Outstanding
400,000	100,000	625,000	0

Trend Factors

July 1, CY6 to July 1 CY7	July 1, CY6 to July 1, CY8	January 1, CY6 to July 1, CY7	January 1, CY6 to July 1, CY8
1.18	1.30	1.24	1.36

	Territory 1	Territory 2
Present Rates		
Class 1 (Differential)	100 (1.00)	200 (2.00)
Class 2 (Differential)	300 (3.00)	600 (6.00)
Collected Earned Premium	700,000	600,000
Policy Year 5 Incurred Losses as of December 31, CY6	360,000	240,000
Earned Exposure Units		
Class 1	5,000	2,000
Class 2	1,000	500

Solution

For each of the two methods, loss ratio and loss cost, the rate change involves the three stages of (i) overall or average rate change, (ii) change in differentials, and (iii) balance back.

Basic Ingredients

We must first calculate the expected losses to be incurred in the future exposure period, trended and developed. We begin by calculating a loss-development factor to apply to the CY5 policy year losses. From the policy year CY4 data, we can see that, at the CY6 reporting (i.e., as of 24 months), the total incurred losses were 500,000 (400,000 + 100,000). One year later (as of 36 months), the total incurred losses were 625,000. These losses are considered fully mature because there are no reserves. Thus, policy year CY4 data exhibited loss development of +25% from first report to fully mature.

Assuming the same loss development for CY5 data, we would use a loss-development factor of 1.25 to develop the CY5 policy year losses to their expected ultimate value.

Next we calculate the trend factor, making the use of the following diagram.

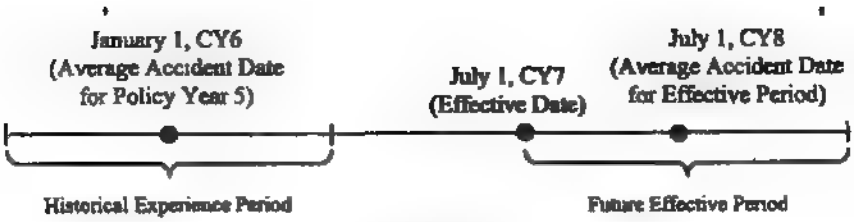


Figure 4.5

The experience data come from policy year 5 with an average accident date of January 1, CY6. The effective date is July 1, CY7 so the future effective period is from July 1, CY7 through June 30, CY9, with an average accident date of July 1, CY8. Thus the required trend factor is the one that goes from January 1, CY6 to July 1, CY8, which is given as 1.36. Then in total we find

$$
\begin{aligned}
&E\,[\text{Dollars of Future Losses (Developed and Trended)}] \\
&\quad = (360{,}000 + 240{,}000)(1.25)(1.36) \\
&\quad = 1{,}020{,}000.
\end{aligned}
$$

Loss Ratio Method

(i) Overall Rate Change

For the loss ratio method the actuary must calculate the earned premium at current rates. The accounting entry for collected earned premium is not the correct denominator, because it could contain earned premiums based on the rates in rate manuals older than the current manual.

$$
\begin{aligned}
&\text{Earned Premium at Current Rates} \\
&\quad = \sum_{i,j} CR_{ij} \cdot e_{ij} \\
&\quad = (100)(5{,}000) + (300)(1{,}000) + (200)(2{,}000) + (600)(500) \\
&\quad = 1{,}500{,}000.
\end{aligned}
$$

This produces

Expected Effective Period Loss Ratio at Current Rates

$$= \frac{1,020,000}{1,500,000} = 0.68,$$

which, based on a permissible loss ratio of 0.60, leads to

$$\text{Indicated Rate Change} = \frac{0.68}{0.60} - 1 = +13.3\%.$$

(ii) Change in Differentials

The given data allow for a territorial differential change analysis but not a class differential change analysis, because loss data by class were not given. We are told that class differentials will remain the same, and are asked to determine the indicated new differentials for Territories 1 and 2. (We must assume the data are 100% credible.)

Territory 1 Earned Premium at Current Rates:

$$(100)(5000) + (300)(1000) = 800,000$$

Territory 2 Earned Premium at Current Rates:

$$(200)(2000) + (600)(500) = 700,000$$

Territory	Existing Differential	Loss Ratio at Current Rates	Indicated Differential
1	1.00	$\frac{360,000}{800,000} = 0.450$	1.00
2	2.00	$\frac{240,000}{700,000} = 0.3429$	$\frac{0.3429}{0.45}(2.00) = 1.5238$

It is irrelevant to this territorial differential analysis whether losses are trended and developed or not, since we are analyzing ratios of loss ratios. Note that as presented, the Territory 1 differential has been left at 1.00, whereas the Territory 2 differential has been reduced from 2.00 to 1.5238. This suggests that the actuary could define the new rates as follows:

	Territory 1	Territory 2
Class 1	113.33	172.70
Class 2	340.00	518.09

If this were done, however, the resulting rate increase would be less than the required 13.3%, due to the off-balance created by the method used to change differentials. This is adjusted in the balance-back step.

(iii) Balance Back

Old Average Differential:

$$\frac{(5000)(1)+(1000)(3)+(2000)(2)+(500)(6)}{8500} = 1.7647$$

New Average Differential:

$$\frac{(5000)(1)+(1000)(3)+(2000)(1.5238)+(500)(4.5714)}{8500} \quad 1.5686$$

The balance-back factor is

$$\frac{\text{Old Average Differential}}{\text{New Average Differential}} = \frac{1.7647}{1.5686} = 1.1250,$$

leading to the following proposed rates:

	Territory 1	Territory 2
Class 1	127.50	194.28
Class 2	382.50	582.85

These proposed rates will result in a 13.3% increase in premium income, as required.

Loss Cost Method

(i) Average Rate Change

We have already calculated:

$$E[\text{Dollars of Future Losses (Developed and Trended)}] = 1{,}020{,}000,$$

from which we find

$$\text{Indicated Loss Cost } = \frac{1{,}020{,}000}{8{,}500} = 120.00$$

and

$$\text{Average Gross Rate } = \frac{120}{PLR} = \frac{120}{0.60} = 200.00$$

(Note that this is the indicated average gross rate, but not the indicated rate for any particular territory or class, which can be determined only when we know the new average differential for our expected book of business.)

(ii) Change in Differentials

To set the new territorial differentials, the actuary normally calculates the average loss costs for Territory 1 and Territory 2, and compares them as follows:

Territory	Existing Differential	Loss Cost	Indicated Differential
1	1.00	$\frac{360{,}000}{6{,}000} = 60$	1.00
2	2.00	$\frac{240{,}000}{2{,}500} = 96$	1.60

This is not the same answer as we got from the loss ratio method. As seen in Example 4.6, the loss cost method normally leads to the correct answer only if all cross-variable distributions are homogeneous, which is not the case here. Recall the following earned exposure unit data:

	Territory 1	Territory 2
Class 1	5,000	2,000
Class 2	1,000	500

In Territory 1, $\frac{5}{6}$ of drivers are Class 1 and $\frac{1}{6}$ are Class 2. In Territory 2, $\frac{4}{5}$ of drivers are Class 1 and $\frac{1}{5}$ are Class 2. To arrive at the correct answer, this heterogeneity of cross-variable distributions must be reflected. One way to accomplish this is to use exposure units that are weighted by their cross-parameter differentials. That is, Class 1 will count as an exposure unit with weight 1.00, but Class 2 will count as an exposure unit with weight 3.00, because of its class differential of 3.00. This leads to the following results:

Territory	Existing Differential	Weighted[4] Units of Exposure	Loss Cost per Weighted Unit of Exposure	Indicated Differential
1	1.00	8,000	$\frac{360{,}000}{8{,}000} = 45.00$	1.00
2	2.00	3,500	$\frac{240{,}000}{3{,}500} = 68.57$	1.5238

(iii) Balance Back

For any risk cell: $Rate(i, j) = (Base\ Rate)(D_i)(D_j)$.

Thus, the actuary determines the gross rate for the base class: Class 1, Territory 1 that will produce the correct manual rates by balancing back for the average indicated differential. Thus

$$\text{Base Gross Rate} = \frac{\text{Average Gross Rate}}{\text{Average Differential}},$$

where the average gross rate is 200 and the average differential is

[4] These are normally called "Base Exposures." For further discussion see: Risk Classification by Robert J. Finger in the textbook *Foundations of Casualty Actuarial Science*, Casualty Actuarial Society, 2001.

$$\frac{(5,000)(1)+(1,000)(3)+(2,000)(1.5238)+(500)(4.5714)}{8,500}=1.5686.$$

This leads to

$$\text{Base Gross Rate} = \frac{200}{1.5686} = 127.50.$$

The resulting manual rates are the same as with the loss ratio method, as expected. A general proof of the equivalence of the manual rates under the loss cost and the loss ratio methods is given in the Appendix. □

EXAMPLE 4.8

Given the following data, show that Exposure Units, adjusted for "heterogeneity" in the cross distributions, are equivalent to Earned Premiums at Current Rate Levels divided by the corresponding Base Rates.

Territory	Current Base Rates	Current Average Rates	Earned Premiums at Current Rates
A	250	200	$1,000,000
B	110	100	1,540,000
C	200	200	800,000

Solution

Earned Premium at Current Rate
$= (\text{Current Average Rates}_i)(\text{Exposures}_i)$

So, $\text{Exposure Units} = \dfrac{\text{Earned Premium at Current Rates}}{\text{Current Average Rates}}$

Territory	Exposure Units
A	$\frac{1,000,000}{200} = 5,000$
B	$\frac{1,540,000}{100} = 15,400$
C	$\frac{800,000}{200} = 4,000$

But if we know that there is heterogeneity in the cross distributions, then we should use Adjusted Exposure Units in the Loss Cost method, not pure Exposure Units.

Comparing current average rates to current base rates, above, we can see that the adjusted Exposure Units for Territories A and B will be smaller (in the ratio $\frac{200}{250}$ and $\frac{100}{110}$) than the pure Exposure Units. Thus we get the following adjusted (for heterogeneity) Exposure Units.

Territory	Exposure Units
A	$5{,}000\frac{(200)}{250} = 4{,}000$
B	$15{,}400\frac{(100)}{110} = 14{,}000$
C	$4{,}000\frac{(200)}{200} = 4{,}000$

But notice that this is the same result as

[Earned Premiums at Current Rates] / [Current Base Rates]

Territory	Earned Premiums at Current Rates	Current Base Rates	Adjusted Exposures
A	$1,000,000	250	4,000
B	1,540,000	110	14,000
C	800,000	200	4,000

Applications of this Example will be seen in Exercises 4.23 and 4.26.

■

4.9 EXERCISES

Section 4.4

4.1 Using the following information, determine the incurred losses for each of the following periods.

(a) Calendar year CY4
(b) Calendar year CY6
(c) Accident year AY5(as reported at 12/31/CY6)

Occurrence #1
Date of occurrence: 12/01/CY4
Date of report: 12/15/CY4
Loss History:

Date	Total Paid-to-Date	Unpaid Loss Reserve	Total Incurred
12/15/CY4	0	500	500
12/31/CY4	500	500	1,000
12/31/CY5	500	500	1,000
01/15/CY6	2,000	0	2,000

Occurrence #2
Date of occurrence: 12/15/CY5
Date of report: 02/01/CY6
Loss History:

Date	Total Paid-to-Date	Unpaid Loss Reserve	Total Incurred
02/01/CY6	1,000	2,000	3,000
12/01/CY6	5,000	0	5,000

Occurrence #3
Date of occurrence: 12/31/CY5
Date of report: 02/15/CY6
Loss History:

Date	Total Paid-to-Date	Unpaid Loss Reserve	Total Incurred
03/15/CY6	0	10,000	10,000
12/31/CY6	0	5,000	5,000

4.2 Given the following information, calculate the total Accident Year AY5 incurred losses as of December 31, CY6.

Claim Number	Year Accident	Amount Paid in CY5	Amount Paid in CY6	Reserve 12/31/CY5	Reserve 12/31/CY6
1	AY4	100	300	1,100	0
2	AY5	200	0	50	200
3	AY5	0	300	150	0
4	AY6	0	100	0	100

Section 4.5

4.3 An incurred-to-earned loss ratio is defined as

Incurred Losses / Earned Premiums.

Given the following information, calculate the CY6 calendar year incurred-to-earned loss ratio.

Premium Written in CY6	100,000
Losses Paid in CY6	90,000
Unearned Premium, 12/31/CY5	50,000
Unearned Premium, 12/31/CY6	40,000
Loss Reserve, 12/31/CY5	160,000
Loss Reserve, 12/31/CY6	140,000

4.4 The XYZ Insurance Company issues both six-month and one-year policies. For CY6 you are given the following information:

Policy Type	Written Premium
Six-month	24,000,000
One-year	120,000,000

Calculate the amount of written premium that would be earned by December 31, CY6, assuming a uniform distribution of policy issues.

Section 4.8

4.5 You have been asked to prepare a rate filing for States A and B for a policy year effective date of January 1, CY7. In State A, policyholders have renewal dates evenly distributed throughout the year, whereas in State B, all policies renew January 1. For each of these two states, to what date would you trend the losses?

4.6 You are developing rates for one-year policies, to be effective August 1, CY6 for one year, based on accident year experience from AY4 and AY5 weighted 40% and 60%, respectively. Your trend factors will be of the form $(1+i)^t$. For the AY5 data, what is t?

4.7 You are the actuary of a small property/casualty company, and have filed for a rate increase in State X. The regulatory board has refused any rate increase, stating that data gathered for the last two years show loss ratios below the permissible loss ratio. How would you reply to the board?

4.8 Some regulators claim there is an overlap in applying both a loss-development factor and a trend factor in setting rates, in that the inflation factor is being double-counted. How would you respond to this claim?

4.9 You are given the following information.

Policy Year	Loss Cost	Weight
PY4	200	40%
PY5	217	60%

The calculated trend factor is based on a ln(Loss Cost) model indicating 10% per year compounded continuously, and the effective date of new rates is November 1, CY7. Policies are one-year policies and rates are to be in effect for one year. Determine the projected loss cost that will be used to determine rates.

4.10 You have calculated the gross rate to be 600 based on the following information:

Fixed expense per exposure	75
Variable expenses (as a % of gross rate)	14%
Profit and contingences (as a % of gross rate)	5%

Calculate the revised gross rate if loss costs increase by 10%.

4.11 Your company writes auto insurance in Ontario. That province has just passed legislation mandating a flat commission expense of \$50 per policy, effective on the date of each company's next rate change. At the present time your average gross premium is 500 per policy, which includes the following percentage loadings.

Commissions	12%
General Expenses	15
Taxes and Licenses	3
Profit and Contingencies	3
	33%

If no other change is indicated, what effect would the legislation have on the average gross premium? Assume all other expenses do not apply to the new commission.

4.12 You are given the following information.

(i) Rate levels are currently adequate for the upcoming policy period.

(ii) The following current expense and loss provisions:

Commissions	20%
General Expenses	8%
Taxes	3%
Profits	5%
Loss and Loss Adjustment	64%

(iii) General expense and profit *dollars* will remain fixed at current levels for the upcoming policy period.

If the existing gross rate is 1000, what should the gross rate be if commissions are reduced to 12%?

4.13 Company A has an expense ratio of 30%, which includes a profit and contingencies margin of 2.5% of the gross premium. Company B discounts all loss payments using a single discount factor of $(1+i)^{-1/2}$ in its pure premium calculation, but includes a profit and contingencies margin of 5% of the gross premium. All other factors, including all other expenses, are the same for both companies.

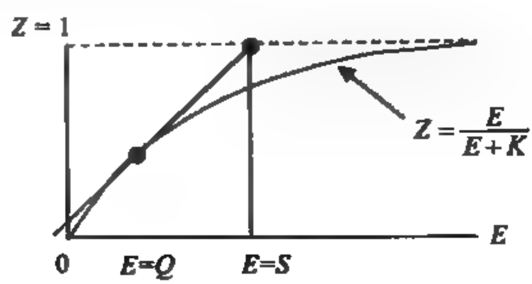
At what rate i will Company A and Company B have equivalent gross premiums?

4.14 Prove that the following formulae satisfy the three properties given in Section 4.7.5 for a credibility factor, Z.

(a) $$Z = \left.\begin{array}{lll} Z=\sqrt{\frac{E}{K}} & \text{for} & E \le K \\ Z=1 & \text{for} & E > K \end{array}\right\} \quad E<0, K>0$$

(b) $$Z = \frac{E}{E+K} \qquad\qquad E>0, K>0$$

4.15 Using the formula $Z=\frac{E}{E+K}$, $K>0$, for credibility, Z can never equal one. Traditionally, actuaries have set $Z=1$ at a ***self-rating point*** $E=S$. Then the actuary determines a point $E=Q$ such that a tangent from the curve $Z=\frac{E}{E+K}$, starting at point $\left(Q, \frac{Q}{K+K}\right)$ will track linearly to the self-rating point where $Z=1$ at $E=S$, as shown in the following diagram. Thus $Z=\frac{E}{E+K}$ for $0 \le E \le Q$, Z lies on the tangent line for $Q \le E \le S$, and $Z=1$ for $E>S$.



Show that

(a) $Q = \frac{1}{2}(S-K)$

(b) $Z = \dfrac{(S-K)^2+4KE}{(S+K)^2}$, for $Q \le E \le S$.

Section 4.9

4.16 You are given the following calendar year earned premium.

Year	Earned Premium
CY4	2,927
CY5	3,301
CY6	3,563

You are also given the following rate changes.

Date	Average Rate Change
July 1, CY2	+10%
July 1, CY4	+8
April 1, CY6	+5

Determine the approximate earned premium at current rates for each of those years. Assume all policies are one year policies.

4.17 Given the following data and assuming 100% credibility for all classes and a uniform expense ratio, find the new indicated differential for Class 2.

Class	Existing Differential	Experience Loss Ratio
1	1.00	0.500
2	1.10	0.580

4.18 You are the chief actuary for the Bonus Insurance Group (BIG), and have been asked to review the proposed multiplicative differentials calculated by one of your actuarial students.

Class	Existing Differential	Loss Ratio	Loss Cost	Recommended Differential
A	1.00	0.500	80.00	1.00
B	1.05	0.520	84.00	1.05
C	1.10	0.510	91.00	1.14
D	1.20	0.490	92.00	1.15
E	1.30	0.540	99.00	1.24
F	1.40	0.510	210.00	1.50
G	1.50	0.520	410.00	1.75

What would you recommend?

4.19 Given the following information, determine the proposed base class rate for Territory 3. The statewide rate level change is to be +7%, and class differentials will not be changed. (Hint: Start by finding the existing base rate for each territory.)

Note: Assume the distribution of cars by class is constant for all territories.

Territory	Earned Exposures	Existing *Average* Rate	Loss Ratio at Current Rates
1	500	190	0.720
2	300	163	0.620
3	200	120	0.770

Class	Differential	Exposure Units
1	1.00	600
2	1.25	100
3	1.50	100
4	1.75	100
5	2.00	100

4.20 You are the pricing actuary for the ABC Insurance Company. Legislation in your home state has mandated that commissions to agents be a constant 50, regardless of class, effective July 1, CY7. Your company has CY5 policy year experience. Your company wishes to combine a rate change with the effective date of the legislated change in commissions. You are given the following information.

CY5 Earned Premium at Current Rates	7,623,251
CY5 Earned Exposure Units	30,493
PY5 Policy Year Incurred Losses (developed to ultimate)	4,192,788

Average Class Differential: 1.25
Effective *Annual* Change in Pure Premium (trend): 9%
CY5 Expenses as Percentages of Gross Premium

Commissions	25%
General Expenses	5
Taxes and Licenses	3
Profit Loading	5

Assuming annual policies and no growth in exposure, what is the new indicated base rate if no taxes or expenses are assigned to the new commission?

4.21 Given the following data, develop the Class B rate for Territory 3. The indicated province wide rate change is to be 3%, and class differentials will not be changed. (Hint: Start by finding the existing base rate for each territory.)

Territory	Earned Exposures	Existing *Average* Rate	Loss Ratio at Current Rates
1	3,000	225	0.700
2	1,500	200	0.660
3	3,500	180	0.720

Class	Exposure in Territory 1	Exposure in Territory 2	Exposure in Territory 3	Current Rates Differentials
A	2,000	800	2,250	1.00
B	150	150	200	1.10
C	600	400	800	0.90
D	100	100	200	1.25
E	150	50	50	2.00
	3,000	1,500	3,500	

4.22 Given the following information, calculate the indicated rate change.

Policy Year	Premium at Current Rates	Ultimate Losses and ALAE	Weight to be given
PY3	2,000,000	1,000,000	0.30
PY4	3,000,000	2,000,000	0.70

Effective Date of Rate Change: 1/1/CY7 on one-year policies
Permissible Loss Ratio: 0.600
Annual Loss Trend: 5%

4.23 You have been asked to make a rate change for one-year policies, effective July 1, CY6, given the following information.

Policy Year	Incurred Losses as of March 31, CY4	March 31, CY5	March 31, CY6
PY1	660,000	693,000	693,000
PY2	800,000	880,000	924,000
PY3		900,000	990,000
PY4			1,000,000

Loss Trends:	January 1, CY5 to July 1, CY6	1.20
	January 1, CY5 to July 1, CY7	1.30
	July 1, CY4 to July 1, CY6	1.25
	July 1, CY4 to January 1, CY7	1.35
	July 1, CY4 to July 1, CY7	1.40

Permissible Loss Ratio: 0.700

Current Average Rates: Territory A 80
Territory B 150

	PY4 Policy Year Experience			
Territory	Exposure Units	Earned Premium	Incurred Losses	Number of Claims
A	10,000	800,000	520,000	1,400
B	8,000	1,000,000	480,000	876

Credibility Factor for Differentials:

$Z = \sqrt{\frac{n}{1{,}082}}$, $0 \le Z \le 1$, where n is number of claims.

Find the new average rate for Territory B.

4.24 You have the following information for your use in pricing an insurance product. The permissible loss ratio is 0.80 and the current province wide base premium is 100.

Existing Territorial Differentials:	Territory 1	1.00
	Territory 2	1.10
Existing Class Differentials:	Class 1	1.00
	Class 2	1.20

CY4 Earned Exposures:

	Territory 1	Territory 2
Class 1	10,000	2,000
Class 2	4,000	2,000

CY4 Earned Premium:

	Territory 1	Territory 2
Class 1	900,000	180,000
Class 2	460,000	250,000

AY4 Incurred Losses (including LAE):

	Territory 1	Territory 2
Class 1	700,000	200,000
Class 2	400,000	270,000

Rates are to become effective February 1, CY6 for 12 months, and all policies are one-year policies. The trend factor is 8% per annum effective.

Accident Year	Incurred Losses at 12/CY2*	12/CY3	12/CY4
AY1	1,000,000	1,050,000	1,050,000
AY2	1,100,000	1,210,000	1,270,500
AY3		1,300,333	1,430,000

Determine the proposed base rate for Territory 2, Class 2 based on the AY4 claims experience.

4.25 Given

Policy Year	Premiums at Current Rates	Ultimate $Loss & ALAE	Weight to be given
PY3	$3,000,000	$1,500,000	30%
PY4	4,500,000	3,000,000	70%

Effective Date for New Rates:
January 1, CY7 on one-year policies
Permissible Loss Ratio = .650
Annual Loss Trend = 6% per annum effective

Calculate the indicated rate change.

4.26

Terr	Current Base Rates	Current Average Rates	Earned Premiums at Current Rates	Incurred $ Loss	Claim Count
A	110	100	$550,000	$330,000	876
B	55	50	770,000	525,000	1,100
C	100	100	400,000	290,000	270

Credibility: $Z = \sqrt{\frac{n}{1,082}}, 0 \leq Z \leq 1,$ where n = # of claims

Find the new Territory differentials:

a) Using the Loss Ratio method
b) Using the Loss Cost method

4.27 Given the following data, indicate the new rates for Classes 1, 2 and 3 if the statewide adopted rate level increase is +7% overall.

Class	Current Relativity	Z	Earned Premiums at Current Rates	Experience Loss
1	1.00	1.00	\$50,000	\$30,000
2	1.25	0.70	20,000	10,560
3	1.50	0.80	30,000	16,200

The existing base rate is \$100 (in Class 1).

INTERMEDIATE TOPICS ○ 5

5.1 INTRODUCTION

Chapters Three and Four introduced the topics of loss reserving and ratemaking. Chapter Five focuses on a few intermediate topics that are not directly addressed in earlier chapters. These topics are individual risk rating, increased limits factors, deductible factors, and reinsurance.

5.2 INDIVIDUAL RISK RATING PLANS

The ratemaking material presented in Chapter Four presents the ratemaking methodology for determination of manual rates. In other words, manual rates are the rates that would be charged to the average member of a homogenous group of individuals with similar risk characteristics. This is the approach used in setting rates for many lines of insurance, such as an individual's automobile insurance policy or homeowners insurance policy. Rates for such policies would only be individual to the extent that the rates reflect the individual's risk characteristics. An individual's specific loss experience is not directly used in the rating calculation, although the occurrence of a prior claim may be a fact that is used to establish an individual's risk characteristics and risk class.

In contrast to manual rating plans, individual risk rating plans are plans in which the insured's premium is determined using the actual loss experience or specific characteristics that are not already reflected in the manual rating. This is the approach used in rating for many commercial lines of business, where establishing homogenous groups for rating may not be possible or where the risk may be large enough that its actual experience may be used in whole or in part to determine its rates. Manual rates may still be used in the rating for such risks when the size of the risk is not large enough to be fully credible.

Individual risk rating is used for ***traditional risk financing*** plans (i.e., insurance) and also ***nontraditional risk financing*** plans (i.e., self-insurance). An entity may have many reasons to self-insure, including but

not limited to: dissatisfaction with its existing insurance coverage or costs, the need for a tailor-made solution, and to have more control over risk financing. Self-insurers may need to allocate the costs of a self-insurance program among the participants of the program, referred to as a ***funding allocation***. When the funding allocation is 100% based on actual losses, there is no risk sharing among the participants. When the funding allocation is 100% based on exposures, there is complete risk sharing. Each self-insurer would need to establish where to balance its desired level of risk sharing to set the allocation.

The primary goal of an individual risk rating plan for an insurer is a more accurate premium determination than the use of manual rates. Such plans provide incentives to insureds to promote risk control and therefore reduce their costs. For self-insureds, the benefit of an individual risk rating plan is that the costs are allocated to participants more accurately and also promote risk control.

The two basic types of individual risk rating plans are prospective and retrospective. ***Prospective rating*** plans use past loss experience to set rates for the future rating period. ***Retrospective rating*** plans use actual experience of the policy period to retroactively determine the final costs for that period.

5.2.1 Prospective Rating Plans

Four prospective rating plans will be described.

Schedule rating is used to incorporate judgment about an insured's risk characteristics that are either not already captured in the historical experience or not reflected adequately in the manual rate. An insured might install a sprinkler system that is expected to reduce losses in the future, for example. As the sprinkler system was just installed, the insured's historical experience would not yet reflect this reduction in expected losses and a modification would be made to adjust the rate to reflect the lower level of future losses expected. Schedule rating is also used for many smaller insureds that may not be large enough to qualify for experience rating.

Schedule rating is usually in the form of a percentage credit or debit for each such risk characteristic, summed together as a single adjustment factor that is applied either before or after experience rating. The

application of schedule rating credits and debits, often involve considerable underwriting judgment.

In prospective ***experience rating*** plans, an experience modification factor is determined by comparing actual loss experience to expected loss experience. This factor is used to modify the manual rate to determine the insured's premium for the prospective policy period.

The basic prospective experience rating formula for the experience modification factor, M, is

$$M = \left(\frac{\text{actual experience}}{\text{expected experience}} \times Z\right) + (1 - Z), \tag{5.1}$$

where Z is a credibility factor.

A value of M less than 1 creates a credit and a value of M greater than 1 creates a debit. Some experience rating plans may also specify minimum and maximum values for M.

There are two basic principles that underlie experience rating plans. First, credibility increases as the size of the insured increases. Second, frequency is a better predictor of future experience than severity.

Composite rating is an administrative tool to assist with the rating of large, complex commercial risks. An initial (or deposit) premium for the prospective rating period is determined by multiplying the composite rate by the estimated composite exposures at the beginning of the policy period. The final premium is then determined after the policy period expires and upon an audit of the final composite exposures. Composite exposures for general liability, for example, might be sales revenue, square footage, number of steel tons manufactured, and for automobile liability might be total mileage driven.

Composite rates may either be based on manual rates adjusted for schedule rating and/or experience modification, or based on the insured's actual experience.

Commercial entities may also purchase insurance with a large deductible, referred to as ***large deductible policies***. While the main goal of small

deductibles is an expense reduction for the insurer as many smaller nuisance claims are not handled, a large deductible serves a different purpose. At a high enough level, the insured would be bearing much of the risk and quite possibly most of the claims would fall within the deductible. As a result, there are several important questions to answer. Who will handle all the claims within the deductible? Additional expense would be expected if the insurer does all of the claims handling. How will the claims within the deductible be paid? In some cases, the insurance company handles all of the claim payments and then seeks reimbursement from the insured. Will the insurance company need to price for credit risk in that circumstance? Does the deductible apply to losses only, or to the sum of losses and ALAE?

The pricing for the large deductible policy will vary depending on the answers to these questions, plus other relevant questions, but fundamentally the pricing is the same as the standard deductible shown in section 5.4.

5.2.2 Retrospective Rating Plans

Under ***retrospective rating*** plans, the actual losses of the current policy are used to determine the premium for that same policy year. As a result, it may take several years before the final premium is known. The incentive to insureds to promote risk control and reduce costs is greater for retrospective plans since the premium is based on actual losses during the policy period. Premiums under such plans are more responsive to change, but can also have greater fluctuations from year to year.

With retrospectively-rated policies, a deposit premium is determined at the beginning of the policy period. The premium is often the same premium determined through prospective experience rating. Retrospective adjustments, either refunds or additional payments, are then made periodically depending on the terms of the agreement.

Retrospective plans may limit losses on a per occurrence basis and also limit losses in aggregate. These plans also may have minimum and maximum premium charges.

5.3 INCREASED LIMITS FACTORS

Chapter Two introduced the topic of policy limits for liability coverages. A policy limit caps the financial obligation of the insurer. Liability coverages are generally written at various policy limits, and pricing for the various policy limits is the focus of this section.

Rates are determined relative to a specific policy limit, referred to as the ***basic limit***. The basic limit is often the minimum policy limit offered to the policyholder. The rates for other policy limits are determined as a multiplicative factor of the basic limit rate. These factors are known as ***increased limits factors*** (ILFs). In essence, the base rate as determined in Chapter Four is calibrated to a common policy limit; the basic limit.

For the purpose of this section, the focus is single limit policies that apply the limit on a per-occurrence basis. The per-occurrence limit is the most an insurer will pay for each occurrence of an insured loss.

In determining the ILFs, it is important to consider the treatment of allocated loss adjustment expenses (ALAE). The insurance policy specifies the treatment of ALAE. Some policies specify that the insurer is responsible for unlimited ALAE while others include ALAE in the policy limit.

EXAMPLE 5.1

An insured purchases a liability policy with a limit of $500,000. Determine the amount the insurer will pay if the insured has a covered loss of $400,000 and ALAE of $200,000. Calculate both for the scenario where ALAE is unlimited and the scenario where ALAE is included in the policy limit.

Solution

1: Unlimited ALAE
Insurer's payment = (Loss limited to $500,000) + (total ALAE)
= $400,000 + $200,000 = $600,000

2: ALAE included in policy limit
Insurer's payment = (Loss + ALAE), limited to $500,000 = $500,000

■

ILFs can be determined using the same methodology presented in Chapter Four, treated as just another rating variable. Many higher limits can have few exposures and few claims and as a result, the experience for many higher limits has higher volatility and lower credibility. This can create problems as the volatility can lead to counterintuitive results. As an example, if the loss experience for a \$5,000,000 policy limit is better than the experience for a \$500,000 limit, a lower rate for the higher limit is implied. This is counterintuitive as the rates for higher limits should always be greater than the rates for lower limits, all other things being equal.

Thus, an alternative approach to the methodology in Chapter Four is generally used for pricing ILF factors, which is based on the expected value of losses (or pure premiums) by limit. The formula for determining the ILF for policy limit L, is given as:

$$ILF_L = \frac{\text{expected losses at limit } L}{\text{expected losses at basic limit}}. \tag{5.2}$$

The expected losses are then derived based on their frequency and severity components, resulting in formula 5.3.

$$ILF_L = \frac{(\text{expected frequency})_L(\text{expected severity})_L}{(\text{expected frequency})_B(\text{expected severity})_B}. \tag{5.3}$$

Two simplifying assumptions are made to further reduce the formula. First, frequency and severity are assumed to be independent. Second, frequency is assumed to not vary by policy limit. Formula 5.4 gives the final form of the ILF.

$$ILF_L = \frac{(\text{expected severity})_L}{(\text{expected severity})_B}. \tag{5.4}$$

Empirically, the ILF formula uses the historical severity, referred to as the limited average severity (LAS).

EXAMPLE 5.2

You are given data for an insurer that sells Personal Automobile Liability policies at the following single limits: \$200,000, \$500,000 and \$1,000,000. Calculate the increased limits factors (ILFs) for \$500,000 and \$1,000,000,

using a basic limit of the \$200,000 and the following historical loss experience for the most recent five accident years. Policy limits include ALAE.

Size of Loss	# of Claims	Ground-Up Total Losses and ALAE
1 - 200,000	1,240	155,124,000
200,001 – 500,000	435	126,150,000
500,001 – 1,000,000	91	65,065,000
Total	1,766	346,339,000

Solution

First we need to determine the limited average severity for each policy limit. To do this, we include only the amount of each loss up to the policy limit. We would, for example, only include the first \$200,000 of losses for each of the 435 claims that have a size of loss between \$200,000 and \$500,000, or \$870,000,000 (\$200,000×425). Also note that losses are described as being ***ground-up***. Ground-up losses are losses from the first dollar of coverage prior to the application of any deductible.

Size of Loss	Losses Capped at \$200,000	Losses Capped at \$500,000	Losses Capped at \$1,000,000
1 – 200,000	155,124,000	155,124,000	155,124,000
200,001 – 500,000	87,000,000	126,150,000	126,150,000
500,001 – 1,000,000	18,200,000	45,500,000	65,065,000
Total	260,324,000	326,774,000	346,339,000

Using the total dollar of losses capped at each limit, we calculate the limited average severity for each limit as follows:

$$LAS(200,000) = \frac{260,324,000}{1,766} = 147,409$$

$$LAS(500,000) = \frac{326,774,000}{1,766} = 185,036$$

$$LAS(1,000,000) = \frac{346,339,000}{1,766} = 196,115$$

Increased limits factors are then calculated using these limited average severities.

$$ILF_{500,000} = \frac{LAS(500,000)}{LAS(200,000)} = \frac{185,036}{147,409} = 1.255$$

$$ILF_{1,000,000} = \frac{LAS(1,000,000)}{LAS(200,000)} = \frac{196,115}{147,409} = 1.330 \qquad \square$$

5.3.1 Data Considerations

There are three practical considerations regarding the data to use for determining increased limits factors.

First, the insurer may not know the total amount of a claim. If an insurer pays the full policy limit of, say, \$500,000 on a claim, what was the full amount of the claim? The insurer may only know the amount it pays, in this case \$500,000. In other words, the data are censored at the policy limit and the insurer does not know the full amount of the loss. The full amount of each claim is needed to properly determine the increased limits factors.

The data in example 5.2 are uncensored data, meaning that we know the full amount of each claim, regardless of policy limit or any applicable deductible. The calculation of the ILFs require uncensored data in practice. Censored data is often the only data available and consequently the actuary needs to determine the ILFs using formula (5.3) with a slight adjustment. When determining the limited average severity for a limit of \$500,000, for example, only data from the policies with a limit of \$500,000 or more are used. Claims from policies with a \$200,000 limit would not be used since only the first \$200,000 of a loss would be known.

The second practical consideration is that the amount of losses may be influenced by the amount of the limit. Attorneys may "target" a claim toward the limit that the defendant has.

Thirdly, adverse or self-selection can occur with policy limits. Policyholders who believe they are more likely to have a larger/(smaller) loss are more likely to purchase policies with relatively higher/(lower) limits.

As a result of these issues, professional judgment is often a key part of the calculation and appropriate selection of the increased limits factors.

5.3.2 Loss Development

When determining increased limits factors using historical data, loss development needs to be considered. The loss development factors as shown in Chapter Three were selected using aggregated claims data. Those factors would include development on known claims as well as a provision for pure IBNR. Applying such factors to an individual claim may not be appropriate.

There are several ways to deal with the issues associated with loss development. Actuaries often use only data from claims that have been closed. As a result, the full amount of each claim is known and no further development will occur. This approach can, however, reduce the volume of data used in the analysis, and as a result some form of adjustment may need to be made when using data that includes both open and closed claims. One approach in selecting loss development factors to be applied to a specific loss limit is to use development factors based on the triangles of only the losses that are limited to that amount.

5.3.3 Loss Trend

When using historical loss experience to calculate empirical ILFs, the influence of inflation also needs to be considered. It is generally assumed that inflation is the same for each claim. In practice, available underwriting limits remain fixed, and it is important, therefore, for the actuary to be aware of the leveraged effect of inflation in each layer. A simple example will illustrate this leveraged effect.

Example 5.3

You are given the following loss amounts for four claims that require a one year inflation trend.

Claim #	Loss
1	35,000
2	75,000
3	95,000
4	110,000

Assuming inflation of 7% on each claim, determine the effect of inflation on the claims in the layer up the basic limit of $100,000 and also the effect of inflation in the layer in excess of the basic limit.

Solution:

Each loss amount is separated into the amount up the basic limit of \$100,000, and the amount in excess of the basic limit.

Claim #	Loss by Layer (before inflation) Total Loss	Basic Limit	Excess Limit	Loss by Layer (with 1 year inflation) Total Loss	Basic Limit	Excess Limit
1	35,000	35,000	0	37,450	37,450	0
2	75,000	75,000	0	80,250	80,250	0
3	95,000	95,000	0	101,650	100,000	1,650
4	110,000	100,000	10,000	117,700	100,000	17,700
Total	315,000	305,000	10,000	337,050	317,700	19,350

While the overall effect of inflation is 7% ($315,000 \times 1.07 = 337,050$), the inflation effect by each layer is leveraged as a result of the basic limit remaining at \$100,000. The inflation effects for the basic limit and excess limit layers are

$$\text{Basic Limit:} \quad \frac{317,700}{305,000} - 1 = 4.2\%,$$

$$\text{Excess Limit:} \quad \frac{19,350}{10,000} - 1 = 93.5\%.$$

❐

The effect of inflation is less than full inflation for the basic limit for two reasons. First, claims that are within 7% of the basic limit will have an increase in that layer less than 7%, such as claim #3. Second, claims that are greater than or equal to the basic limit will see no increase due to inflation in the basic layer, such as claim #4.

In a similar way, the effect of inflation tends to be greater than full inflation in the excess limit for two reasons. First, some claims may not have penetrated the excess layer before inflation but do penetrate the excess layer as a result of inflation, such as claim #3. Second, the entire amount of the increase is in the excess layer for those claims that were higher than the excess limit prior to inflation, such as claim #4.

5.3.4 Risk Load

The premium as derived in Chapter Four is comprised of the expected pure premium, an expense provision, and a provision for profit and contingencies. The profit and contingencies provision is intended to compensate appropriately for the risk the insurer assumes. An additional provision, or risk load, is often considered in increased limits pricing for the added risk associated with the additional uncertainty in estimating expected losses for higher limits.

We can modify formula 5.2 to include a provision for risk load into the ILF calculation.

$$ILF_L = \frac{(\text{expected losses at limit } L) + \text{RL}_{(L)}}{(\text{expected losses at basic limit}) + \text{RL}_{(B)}}. \qquad (5.5)$$

Risk load determination is beyond the scope of this textbook. An example will illustrate the risk load in the ILF calculation.

Example 5.4

You are given the same loss data used in Example 5.2. All claims are closed (i.e., no LDF needed) and have been trended to current cost level. Assume policy limits include ALAE. The risk charge for each limit is determined to be the square of the limited average severity divided by 500,000. The basic limit is $200,000. Calculate increased limits factors for policy limits $500,000 and $1,000,000, including the risk charge.

Size of Loss	# of Claims	Ground-Up Total Losses incl. ALAE
1 – 200,000	1,240	155,124,000
200,001 – 500,000	435	126,150,000
500,001 – 1,000,000	91	65,065,000
Total	1,766	346,339,000

Solution

From Example 5.2, we determined the limited average severities to be $147,409, $185,036 and $196,115 for policy limits $200,000, $500,000 and $1,000,000, respectively. The risk charge is calculated to be the square of the limited average severity, divided by 500,000.

Limit	LAS(l)	Risk Charge	Sum of LAS(l) & Risk Charge	ILF
200,000	147,409	43,459	190,868	1.00
500,000	185,036	68,477	253,513	1.33
1,000,000	196,115	76,922	273,037	1.43

For example, the risk charge for the $500,000 limit is given as

$$\frac{185{,}036^2}{500{,}000}=68{,}477\,,$$

and the ILF(500,000) is determined to be

$$\frac{253{,}513}{190{,}868}=1.33\,.$$ □

In comparing the ILFs with the risk charge to the ILFs with no risk charge (see Example 5.2), we notice that the factors including the risk charge are higher, representing the additional charge to reflect the additional uncertainty associated with the higher limits.

5.3.5 Expenses

As covered in Chapter Four, the expenses loaded into premium calculations should be divided into fixed and variable. An insurance company's underwriting cost and policy issue expense do not tend to vary by policy limit. Claims handling expenses (i.e., unallocated (ULAE) and/or allocated adjustment expenses (ALAE)) may also not vary much by policy limit. While it is often the case that a larger claim will take a longer time to settle and therefore cost more in adjustment expenses, the expense is not necessarily directly proportional to the size of the claim. Nonetheless, ILFs calculated in practice usually make the simplifying assumption that all expenses are premium variable and do not vary by policy limit.

5.3.6 Loss Distributions

Although ILFs are typically determined empirically in practice, formula 5.2 can be applied using a fitted loss distribution. This approach has the benefit of a consistent structure for use in determining ILFs. The actuary can fit a theoretical distribution to the actual loss data to smooth the random fluctuations in the data. Common distributions include Pareto, truncated

Pareto, and lognormal. The actual loss data should be adjusted as discussed earlier in this section prior to fitting to a theoretical distribution.

We know if we assume that frequency does not vary by limit, we only need the loss size distribution to determine ILFs. If $f(x)$ represents a continuous distribution of losses of size x, then the formula for LAS for losses at limit L is given as:

$$LAS(L) = \int_0^L xf(x)dx + L\int_L^\infty f(x)dx. \tag{5.6}$$

Formula 5.6 is the sum of total expected losses up to L, plus L multiplied by the probability that a loss will exceed L.

The increased limit factor for losses limited at L, relative to losses limited to the basic limit, B, is then:

$$ILF_L = \frac{\int_0^L xf(x)dx + L\int_L^\infty f(x)dx}{\int_0^B xf(x)dx + B\int_B^\infty f(x)dx}. \tag{5.7}$$

5.4 Deductible Pricing

Chapter Two provided background on deductibles, how they work and how they are used in insurance. This section will explain how one prices for different deductibles. Similar to increased limits pricing, a rate is determined for the base level deductible, B, and rates for other deductible options are determined as a factor of the base level deductible rate.

5.4.1 Loss Elimination Ratios

Similar to the methodology for pricing increased limits factors, an alternative approach to the methodology in Chapter Four is generally used for pricing deductible factors. This approach is based on the expected value of losses by deductible. The formula for determining the indicated deductible relativity for policies with deductible D, is given as:

$$\text{Indicated deductible relativity}_D = \frac{(\text{expected losses})_D}{(\text{expected losses})_B}. \tag{5.8}$$

A loss elimination ratio approach is often used to determine the deductible relativities. The ***loss elimination ratio*** (LER) is defined to be the ratio of the expected losses eliminated to the total expected losses at the base level deductible. In other words, the LER is the difference between the expected losses at the base level deductible and the expected losses at deductible D, divided by the expected losses at the base level deductible.

$$LER_D = \frac{(\text{expected losses})_B - (\text{expected losses})_D}{(\text{expected losses})_B}. \tag{5.9}$$

The indicated deductible relativity can be expressed in terms of the LER, as follows:

$$\text{Indicated deductible relativity}_D = 1 - LER. \tag{5.10}$$

The above formulas assume that expenses and the profit and contingencies loading are premium variable, and therefore, do not impact the deductible relativity calculations.

A simple example will illustrate the application of the LER method.

EXAMPLE 5.5

You are given the following loss data for an automobile collision coverage. The insurer offers the following deductible options: no deductible, $500 (base level), and $1,000.

Size of Loss	# of Claims	Ground-Up Total Losses incl. ALAE
1 – 500	1,240	396,800
501 – 1,000	1,080	831,600
1,001 or greater	2,180	9,352,200
Total	4,500	10,580,600

Calculate the LER for a deductible of $1,000 and the indicated $1,000 deductible relativity. Assume the base level deductible is $500.

Solution:

First, we will need to determine the expected losses at each of the \$500 and \$1,000 deductible levels. We can do this by determining the dollars of loss that would be eliminated at each deductible level, and subtract those values from the total ground-up losses. Note that the total ground-up losses are the amount of losses expected at a \$0 deductible level.

Size of Loss	# of Claims	Total Ground-Up Losses	Losses Eliminated at \$500 Ded	Losses Eliminated at \$1,000 Ded
1 – 500	1,240	396,800	396,800	396,800
501 – 1,000	1,080	831,600	540,000	831,600
1,001 or greater	2,180	9,352,200	1,090,000	2,180,000
Total	4,500	10,580,600	2,026,800	3,408,400

At the \$500 deductible level, for example, we would completely eliminate the losses in the 1 – 500 size of loss range since all losses in this range are \$500 or less. We would only eliminate the first \$500 of losses in each of the higher size of loss ranges (e.g. $1{,}080\times500 = 540{,}000$).

Using the losses eliminated at each deductible amount, we can determine the total expected losses to be paid at each deductible.

$$(\text{expected losses})_{500} = 10{,}580{,}600 - 2{,}026{,}800 = 8{,}553{,}800$$
$$(\text{expected losses})_{1{,}000} = 10{,}580{,}600 - 3{,}408{,}400 = 7{,}172{,}200$$

The LER and indicated deductible relativity are then calculated as follows:

$$LER_{1{,}000} = \frac{8{,}553{,}800 - 7{,}172{,}200}{8{,}553{,}800} = 0.162$$

$$\text{Indicated deductible relativity}_{1{,}000} = 1 - 0.162 = 0.838.$$

Note that the indicated deductible relativity can also be calculated directly using formula 5.8. ❐

5.4.2 Data Considerations

There are several considerations regarding the data to use for determining deductible relativities.

First, the insurer may not know the ground-up losses for every claim. The data used for Example 5.5 includes the ground-up losses for all claims. In practice however, the insurer will often not be aware of every claim if the loss is less than the deductible. An insured may not report a claim that is for an amount clearly less than the deductible. As a result, data from policies with deductibles higher than the deductible factor being priced cannot be used to calculate that relativity. Data from policies with a \$1,000 deductible, for example, cannot be used to determine the relativity for \$500 deductible policies as the \$1,000 deductible policies may have had some losses for amounts between \$500 and \$1,000 of which the insurer is not aware.

Another data consideration is an issue with incomplete data due to policyholders not reporting claims for losses that are greater than the deductible. An insured with a \$1,000 deductible may, for example, decide not to report a claim where the loss is \$1,200. The policyholder may have concerns that the premium increase they are likely to see as a result of having a claim, may exceed the \$200 of indemnity payment they would receive by reporting the claim. As a result, the insurer may not be aware of all loss amounts greater than the deductible.

As with ILFs, there are similar concerns of adverse or self-selection with deductible pricing. Lower/(higher) risk insureds tend to select higher/(lower) deductibles because they expect an overall lower/(higher) claims frequency. Since the LER approach would not reflect such self-selection, those who select higher/(lower) deductibles may be more/(less) profitable to the insurer. A comparison of the indicated deductible relativities using the LER approach to the indicated deductible relativities using the approach from Chapter Four might help give insight into the possibility of self-selection.

Finally, consideration should be given for the situation in which the premium savings for selecting a higher deductible is more than the increase in the deductible. Consider a case, for example, for which the base level deductible is \$500, the \$1,000 deductible factor is 0.82, and the policyholder's base premium is \$3,000. If a \$1,000 deductible is selected, the premium savings for the individual would be \$540 [\$3000×(1 – 0.82)].

This savings is greater than the additional increase the policyholder would pay in the event of a loss, \$500, implying that the policyholder would always select the higher deductible because of the savings, assuming of course they do not expect to have more than one claim during the policy period.

5.4.3 Loss Development And Trend

The same comments regarding loss development and trend that were made for increased limits factors can be made for deductible relativities.

5.4.4 Loss Distributions

In the same way that a fitted loss distribution can be used to determine increased limits factors, we can use a fitted loss distribution to determine deductible relativities.

Assuming that the function $f(x)$ represents the continuous function of losses of size x, then the dollars of loss expected to be eliminated by the application of deductible D, are given by the following expression:

$$\int_0^D xf(x)dx + D\int_D^\infty f(x)dx.$$

The LER is then determined by dividing the expression for the losses eliminated by the total expected unlimited losses.

$$LER = \frac{\int_0^D xf(x)dx + D\int_D^\infty f(x)dx}{\int_0^\infty xf(x)dx}. \tag{5.11}$$

5.5 Reinsurance

Recall from Chapter One that an individual can transfer risk, or variability of possible economic outcomes, to an insurance company. As the number of independent individuals insured by the company increases, there is greater certainty of possible outcomes. In this way, the insurer can better manage the overall risk by all such individuals versus what can be done by any one individual separately. The broader the spread of risk the insurer has, the better it can manage the total risk. Consider, for example, the situation in which a company underwrites 10 tenant insurance policies that cover contents damage from a peril such as fire. There is better spread of risk for

the insurer to insure 10 individuals living in different buildings than all 10 individuals living in the same building. With all 10 individuals living in the same building, there is a much higher probability of multiple losses from one loss event. There are certain circumstances where a company desires to spread the risks it accepts from individuals beyond what it covers by itself. This is where reinsurance is used.

Reinsurance can be described as insurance for insurance companies. The reinsurance contract is an insurance contract between the primary insurer and the reinsurer. The reinsurer agrees to indemnify the insurance company (the ***cedant***) on all or part of the risks that the insurance company writes (insures). In reinsurance terms, the primary insurer ***cedes*** all or part of the risk to the reinsurer, and we say that the reinsurer ***assumes*** all or part of the risk. The reason that primary insurers rely on reinsurance is somewhat similar to the reason individuals transfer risk to insurance companies, in that the underlying risk to too large relative to their risk appetite or tolerance. Even though a primary insurer may transfer part of the risk underlying its insurance portfolio to a reinsurer, the primary insurer still maintains the financial obligation from the originally issued policy. In other words, if a reinsurer is unable to meet its financial obligation to the primary insurer, the primary insurer is still obligated to satisfy the terms of the originally issued policy (i.e., indemnify the policyholder).

Insurers may purchase reinsurance from one or more reinsurance companies. Likewise, reinsurers may also reduce the amount of reinsurance they assume by purchasing reinsurance coverage from other reinsurers, which is referred to as ***retrocession***.

5.5.1 Functions Of Reinsurance

There are many functions of reinsurance, including but not limited to:

(1) Capital relief – An insurer must allocate capital to support the policy risks it writes, but an insurer does not need to allocate capital to support the risk that it transfers to a reinsurer. An insurer can thus reduce its capital needs on its balance sheet by removing risks through reinsurance.

(2) Increase underwriting capacity – An insurer can increase its capacity to write risks that are either too large or too risky to write on its own by transferring those loss exposures, in whole or in part, to a reinsurer.

(3) Catastrophe protection – Reinsurance can protect against significant fluctuations in an insurer's underwriting results that can arise from catastrophic events.

(4) Stabilize loss experience – Reinsurance can protect against fluctuations in an insurer's underwriting results that can arise from non-catastrophic events, such as the occurrence of several large losses.

(5) Risk concentration – A primary insurer may have concerns about its risk concentration, such as too high a concentration in a specific type of risk or too high a concentration in one geographical area.

(6) Technical expertise – A reinsurer normally has greater experience with such things as pricing higher limits policies and claims handling of uncommon and large claims.

(7) Withdrawal – A primary insurer can use reinsurance if it wants to withdraw from a market or a particular line of business by transferring all of the loss exposures to the reinsurer.

5.5.2 Types Of Reinsurance

Reinsurance contracts are typically categorized as either ***treaty*** or ***facultative*** arrangements. Treaty reinsurance arrangements are contracts under which the primary insurer and reinsurer agree to the specific line(s) of business up front that will be ceded to the reinsurer. Facultative reinsurance arrangements are typically used to cover large, complex individual commercial risks. The primary insurer selects each exposure to cede to the reinsurer and the reinsurer then individually underwrites, and therefore accepts or rejects, each risk.

The terms of the reinsurance contract will also define how the reinsurer shares in the risk of the primary insurer. One way that the reinsurer shares in the risk of the primary insurer is ***pro rata*** (or ***proportional***) reinsurance which is a contract in which both the primary insurer and reinsurer share the amounts of insurance, premium, and losses. The share is either defined as ***quota share***, where each shares a fixed percentage of the total risk, or as ***surplus share***, where each shares a fixed percentage of the risk that exceeds the primary insurer's net retention.

In addition, the reinsurer pays the primary insurer a ceding commission. The primary insurer incurs expenses to issue the policies and handle the claims. The ceding commission is essentially a reimbursement for these expenses for the portion of the policies being ceded.

An example of quota share reinsurance is a contract under which the insurer/reinsurer pays 80%/20% respectively of each claim regardless of size. The reinsurer would then receive 20% of the premium and it would pay 20% of all losses including ALAE.

An example of surplus share reinsurance is a contract for the reinsurer to pay 50% of the per occurrence losses excess of $500,000. In such a situation, the primary insurer's ***retention*** is $500,000, meaning that the insurer retains 100% of a loss up to $500,000. For any loss that exceeds $500,000, the primary insurer and reinsurance would each share 50% of the loss beyond $500,000.

A second method with which the reinsurer can share in the risk of the primary insurer is referred to as ***excess of loss*** reinsurance. Such a contract indemnifies the primary insurer for losses that exceed some specified amount (the primary insurer's retention, also known as the reinsurer's ***attachment point***). Excess of loss reinsurance can cover risks on several bases:

- *Per risk* excess of loss covers all losses resulting from one covered claim or occurrence for a single policy.
- *Per occurrence* excess of loss covers all losses resulting from one covered claim or occurrence for all covered policies. Catastrophe excess of loss polices are on a per occurrence basis.
- *Aggregate, or Stop Loss,* excess of loss covers the aggregate of all covered losses over a specified period, typically one year.

Finite, or nontraditional, reinsurance covers are beyond the scope of this text.

5.5.3 Reinsurance Reserving

As explained in earlier sections, the reinsurance transaction reduces the amount of risk carried by the primary insurer. As a result for financial reporting purposes, a primary insurer's total loss reserves can be reduced to reflect the amount of risk that has been transferred to the reinsurer. Recall also that the primary insurer is ultimately responsible for the financial obligation of all of its policies. The primary insurer will thus need to adjust its loss reserves for uncollectable reinsurance. The accounting framework for the primary insurer's financial statements is beyond the scope of this text.

The reinsurer can use many of the same methods for determining its loss reserves that a primary insurer uses; there are, however, several issues that make the exercise more difficult, including but not limited to:

(1) Because we are typically dealing with larger claims, the time lag when claims are reported is typically longer with reinsurance, adding additional uncertainty in estimating ultimate losses.
(2) Many reinsurance contracts are unique and therefore the data source used in loss reserving has many heterogeneous, rather than homogeneous, risks. It is more difficult for the actuary to predict future development patterns with heterogeneous risks due to the lack of statistically-credible loss experience.
(3) In many cases, the reinsurer may only receive summary claims data from the primary insurer, as is common with proportional reinsurance.

The reinsurer's loss reserves would typically include the following components:

(1) Case reserves from the ceding company,
(2) Additional case reserves over and above the ceding company case reserves that the reinsurer's claims department determines,
(3) A provision for development on the known case reserves from (1) and (2),
(4) An estimate for pure IBNR, and
(5) A loading for risk.

Where applicable, there may also be consideration for discounting of loss reserves, which is beyond the scope of this text.

5.5.4 Reinsurance Pricing

The fundamentals of reinsurance pricing are essentially the same as the fundamentals of pricing primary insurance. The pricing of reinsurance covers is understandably more uncertain than for primary insurance, meaning that the reinsurer's underwriting results involves higher variability. There are several reasons for this higher variability, including but not limited to:

(1) The data used for reinsurance pricing often involves non homogeneous risks as the coverage terms are usually very individualized for each contract.

(2) Reinsurance typically covers risks, or the part of a risk, that have lower frequency and higher severity.

(3) The time delay between the occurrence, the reporting and the settlement of a claim is often much longer than for primary covers.

(4) There is greater uncertainty in the claim severity inflation above an excess cover attachment point.

(5) The longer tail nature of many reinsurance coverages contributes to the uncertainty in estimating IBNR and development on case reserves.

There are a variety of methods for pricing reinsurance for the various types of reinsurance covers that exist. We will look at a few examples of reinsurance pricing to illustrate the reinsurance pricing methods.

Proportional Treaty Pricing

In pricing proportional treaties, the reinsurer would need to estimate the expected loss ratio by collecting historical data, excluding catastrophic experience, adjusting the experience to ultimate values, selecting the non-catastrophe expected loss ratio, and then add a loading for catastrophe exposure. The combined ratio is then determined by taking the expected loss ratio and adding the ceding commission rate and other expense provisions. The reinsurer would then evaluate whether this combined ratio yields the desired target profit. If it does not result in the required profit, the reinsurer would then adjust the ceding commission rate accordingly. The adjustment may often involve using a ceding commission rate that varies depending on the loss ratio, also known as a sliding scale commission.

Excess of Loss Treaty Pricing

For excess treaties, the treaty premium is typically determined as a percentage of the primary company (subject) premium. There are two different approaches used by the reinsurer to determine the treaty premium.

Experience Rating

Experience rating assumes that the primary company's historical experience is the best predictor of future experience. Ultimate losses in the layer of coverage are estimated and divided by the subject premium to determine the loss ratio. Losses must first be adjusted for inflation prior to determining the amount of the loss that falls in the layer of coverage. Losses are then developed to ultimate levels and divided by the subject premium adjusted to current rate levels.

EXAMPLE 5.6

You are given the following historical data for five claims with losses greater than the attachment point. The losses have already been trended for inflation to future expected levels.

Date of Accident	Trended Losses and ALAE
4/10/AY1	650,100
2/4/AY2	2,300,000
9/10/AY2	1,240,400
3/1/AY3	984,000
6/15/AY4	1,940,000

In addition, the following subject premiums, adjusted to current rate levels, and loss development factors, are given.

Accident Year	Subject Premium On-Level	Loss Development Factors
AY1	6,526,000	1.00
AY2	6,852,000	1.05
AY3	7,195,000	1.15
AY4	7,555,000	1.40

The excess of loss treaty covers losses and ALAE above the attachment point of $500,000, to a limit of $1,500,000. Calculate the loss ratio for this treaty.

Solution:

First, we will need to determine the amount of each loss that falls within the treaty layer (i.e. the losses above the attachment point, not exceeding the limit).

Date of Accident	Trended Loss	Loss in Treaty Layer
4/10/AY1	650,100	150,100
2/4/AY2	2,300,000	1,500,000
9/10/AY2	1,240,400	740,400
3/1/AY3	984,000	484,000
6/15/AY4	1,940,000	1,440,000

We can now group the losses by accident year, develop the losses to ultimate values, and calculate the loss ratio.

Accident Year	Subject Premium On-Level	Trended Losses in Layer	LDF	Trended Ultimate Losses in Layer	Loss Ratio
AY1	6,526,000	150,100	1.00	150,100	2.3%
AY2	6,852,000	2,240,400	1.05	2,352,420	34.3%
AY3	7,195,000	484,000	1.15	556,600	7.7%
AY4	7,555,000	1,440,000	1.40	2,016,000	26.7%
Total	28,128,000	4,314,500		5,075,120	18.0%

In this case, we determined the loss ratio to be 18% of the subject premium. Prior to quoting a price for the excess of loss treaty, the reinsurer would include a provision for its expenses and profit provision. ❐

Exposure Rating

The exposure rating approach estimates losses within the treaty layer by using a claim severity distribution based on industry data. Increased limits factors are calculated and used to estimate the losses within the treaty layer, but simulation approaches can also be used. The advantage to the exposure rating approach is that instead of assuming past experience is the best predictor of future experience, the current risk profile is modeled to estimate the expected losses that will fall within the layer of coverage.

An example will help illustrate the application of this approach.

EXAMPLE 5.7

A primary insurer writes $20 million (in premiums) of automobile liability insurance. The highest limit that policyholders can purchase is $5,000,000. In order to stabilize its underwriting results, the company wants to reinsure all losses in excess of $1,000,000. For simplicity, assume loss expenses are included in the limits of coverage. The expected loss ratio is 70%, and the increased limit factor (ILF) table based on industry data is (ILFs have not been loaded for risk or expenses):

Limit	ILF
$ 500,000	1.00
$1,000,000	1.15
$2,000,000	1.35
$5,000,000	1.60

Calculate the expected losses in the reinsurance layer (ceded losses) using an exposure rating approach.

Solution

Total expected losses are estimated to be $20,000,000×0.70 = $14,000,000.

The reinsurance will be for the layer between $1,000,000 and $5,000,000, referred to as $4,000,000 excess of $1,000,000.

We can use the ILFs to estimate the losses within the treaty layer. This is done by recognizing that the inverse of the ILFs represent the cumulative limited loss distribution. We know that the highest limit that can be purchased is $5,000,000. Therefore, a limit of $5,000,000 is assumed to represent a cumulative limited loss distribution of 1.0, or 100%. We can re-index the ILFs by dividing by the ILF at $5,000,000 to estimate the cumulative limited loss distribution.

Limit	Cumulative Limited Loss Distribution
$ 500,000	0.625
$1,000,000	0.719
$2,000,000	0.844
$5,000,000	1.000

For example, $1.15 \div 1.60 = 0.719$.

The percent of losses within the treaty layer can be estimated by taking the difference between the cumulative limited loss distribution values at $5,000,000 and $1,000,000, or

$$1.000 - 0.791 = 0.281.$$

This means that 28.1% of the total losses from the primary policies are estimated to fall in the layer $4,000,000 excess of $1,000,000. Since the total expected losses are estimated to be $14,000,000, the losses in the reinsurance layer are estimated to be:

$$\$14,000,000 \times 0.281 = \$3,934,000.$$ ❐

Catastrophe Reinsurance Treaty Pricing

Catastrophe models are typically used to price for reinsurance catastrophe covers by layer. These models simulate the covered hazards such as hurricanes or earthquakes, calculate the intensity for the property risks within a portfolio, and estimate the damage for those properties that are affected by an event that is insured by the ceding company. One of the important considerations for pricing such risks is the possibility of a demand surge. A demand surge is a sudden increase in the cost of materials, services and labor that is caused by a significant increase in demand that follows a catastrophe.

5.6 EXERCISES

Section 5.2

5.1 A commercial business is installing a sprinkler system in its warehouse to reduce possible damages from fire claims. Explain the difference between how the reduction in losses would be reflected in schedule rating versus experience rating.

5.2 You are determining the premium for a composite rating plan and have estimated the composite rate for the upcoming policy period (CY5) to be $42.40 per $1,000 of payroll. The estimated payroll for CY5 is $9,750,000 and the audited payroll for CY5 is $10,105,000. Determine the composite premium.

5.3 An experience rating plan uses a two-year loss ratio weighted by manual premium compared to the expected loss ratio of 60%. The following historical manual premium and developed losses are given below, as well as the formula for credibility.

Year	Manual Premium	Developed Losses
CY2	250,000	125,000
CY3	270,400	145,600

$$Z = \frac{\text{Manual Premium}}{500{,}000 + \text{Manual Premium}}.$$

Calculate the experience rated premium for CY4, assuming the CY4 manual premium is $281,000.

Section 5.3

5.4 You are given the following claims information for a liability coverage. The basic limit is \$200,000 and all losses include ALAE.

Size of Loss	# of Claims	Ground-Up Losses
1 – 200,000	384	47,001,600
200,001 – 500,000	250	81,050,000
500,001 – 1,000,000	140	97,076,000
1,000,001 – 2,000,000	53	75,737,000
2,000,001 – 5,000,000	15	51,750,000
Total	842	352,614,600

Calculate the ILFs for policy limits 1,000,000 and 2,000,000.

5.5 You are given the following probability loss distribution for a liability coverage and the average loss within each size of loss interval.

Size of Loss	Cumulative Probability	Average Loss in Interval
1 – 100,000	0.52	72,500
100,001 – 200,000	0.71	142,200
200,001 – 500,000	0.86	378,900
500,001 – 1,000,000	0.93	712,400
1,000,001 – 1,0000,000	1.00	2,970,000

(a) Calculate the ILF for \$1 million limit assuming the basic limit is \$200,000, with no expenses and no risk load.

(b) Calculate the ILF for \$1 million limit assuming the basic limit is \$200,000, with no expenses, but with a risk load determined to be the average loss size squared divided by 2,000,000.

Section 5.4

5.6 You are given the ground-up losses for the following 5 claims.

Claim #	Ground-up Loss
1	400
2	5,300
3	10,500
4	15,800
5	23,700
Total	55,700

Calculate the indicated relativity for a \$500 and \$1,000 deductible assuming that the base level deductible is \$0.

5.7 You are given the following loss data where you know the ground-up losses for all claims.

Size of Loss	# of Claims	Ground-Up Losses
1 – 500	840	285,600
501 – 1,000	1,260	945,000
1,000 – 2,000	920	1,297,200
2,001 or greater	2,180	8,611,000
Total	5,200	11,138,800

Calculate the indicated relativities for a \$0 deductible and a \$1,000 deductible. Assume the base level deductible is \$500.

5.8 You are pricing deductible factors for windshield replacement coverage. You have determined that the function $f(x)=\frac{x}{500{,}000}$ represents the continuous function of losses of size x, for $0 \le x \le 1{,}000$ (no windshield costs more than \$1,000). Calculate the indicated deductible relativity by moving to a \$500 deductible from the current \$250 deductible.

Section 5.5

5.9 Given the following ILFs, calculate the percent of total losses expected to be paid by the reinsurer if the reinsurer layer was $500,000 excess of $500,000. The maximum policy limit is $2 million.

Limit	ILF
250,000	0.90
500,000	1.00
1,000,000	1.15
2,000,000	1.30

5.10 A reinsurer covers all claims that exceed the attachment point of $500,000. You are given the following information for the only four claims that occurred.

Claim File ID	Amount Paid on Claim
1	250,000
2	495,000
3	540,000
4	750,000

Calculate the percentage increase in claims paid assuming that inflation of 7% affects every claim.

(a) For the primary insurance company,
(b) For the reinsurer.

5.11 An actuary is valuing loss reserves for excess of loss treaties and is combining data from policies with treaties with an attachment point of $500,000 with data from policies with treaties with an attachment point of $2,000,000. Explain whether combining these data sources is appropriate or not.

5.12 An insurer wants to stabilize its net loss ratio. Recommend a type of reinsurance for the company to purchase.

5.13 A primary insurance company has a reinsurance treaty that contains a ceding commission on a sliding scale. The ceding commission will be paid in full if the loss ratio is 60% or less, and will be 0 if the loss ratio is 80% or higher. For a loss ratio between 60% and 80% the ceding commission will be paid out as a pro rata percentage (0% to 100%). A loss ratio of 75% yields a ceding commission of $500,000. Calculate the ceding commission if the loss ratio is 65%.

5.14 You are given the following data for an insurer:

Policy	Premium	Policy Limit	Loss
1	$12,000	$200,000	$150,000
2	3,000	50,000	50,000
3	30,000	500,000	300,000

All policies are covered by a 60% quota share treaty.

a) Calculate the retained premium for the primary insurer.
b) Calculate the retained losses for the primary insurer.

5.15 A primary insurance company has a $100,000,000 retention limit. You are given the following information regarding the coverage provided by a catastrophe reinsurance treaty (in $000,000) (i.e. amount paid by reinsurer):

Layer 1:	85% of 100 excess of 100
Layer 2:	90% of 100 excess of 200
Layer 3:	95% of 300 excess of 300

Given a catastrophic loss of $450,000,000, calculate the total loss retained by the primary insurer.

Appendix

Equivalence of Loss Ratio and Loss Cost Methods of Determining Classification Differentials

We begin by defining the following symbols:

ℓ_{ijk}:	Dollars of Incurred Losses for rate cell (i,j,k)
e_{ijk}:	Units of Earned Exposure for rate cell (i,j,k)
CR_{ijk}:	Current Manual Rate for rate cell (i,j,k)
PLR:	Permissible Loss Ratio

Let us first determine an algebraic expression for what occurs in the setting of classification differentials. Without loss of generality, we limit ourselves to an example with only two classification parameters, class and territory. Thus we have two vectors of differentials,

$$x_i,\ i = 1, 2, \ldots, n$$

for the class parameter, and

$$y_j,\ j = 1, 2, \ldots, m$$

for the territory parameter. Assume there is a base class, B, such that $x_B = y_B = 1.000$. The current rate for the base class is denoted CR_B.

Consider a rate manual produced by the base rate CR_B and the two vectors of relativities x_i and y_j. This will produce a matrix of $m \times n$ rates. Consider, again without loss of generality, that we are calculating the new value of x_k, the differential for Class k. We can think of Class k as occupying the k^{th} row of our rate manual matrix.

Loss Cost Method

The loss cost for Class *k*, adjusted for heterogeneity, is

$$LC_k = \frac{\sum_j \ell_{kj}}{\sum_j e_{kj} \cdot y_j}, \tag{A.1}$$

and the loss cost for the base class, adjusted for heterogeneity, is

$$LC_B = \frac{\sum_j \ell_{Bj}}{\sum_j e_{Bj} \cdot y_j}. \tag{A.2}$$

Then the new differential is

$$x_k^* = \frac{\sum_j \ell_{jk}}{\sum_j e_{kj} \cdot y_j} \bigg/ \frac{\sum_j \ell_{Bj}}{\sum_j e_{Bj} \cdot y_j} = \frac{\sum_j \ell_{kj}}{\sum_j e_{kj} \cdot y_j} \cdot \frac{\sum_j e_{Bj} \cdot y_j}{\sum_j \ell_{Bj}}. \tag{A.3}$$

Loss Ratio Method

The loss ratio for Class *k* is

$$LR_k = \frac{\sum_j \ell_{kj}}{\sum_j e_{kj} \cdot CR_{kj}}, \tag{A.4}$$

and the loss ratio for the base class is

$$LR_B = \frac{\sum_j \ell_{Bj}}{\sum_j e_{Bj} \cdot CR_{Bj}}. \tag{A.5}$$

Then the adjustment factor is

$$\frac{\sum_j \ell_{jk}}{\sum_j e_{kj} \cdot CR_{kj}} \Bigg/ \frac{\sum_j \ell_{Bj}}{\sum_j e_{Bj} \cdot CR_{Bj}} = \frac{\sum_j \ell_{kj}}{\sum_j e_{kj} \cdot CR_{kj}} \cdot \frac{\sum_j e_{Bj} \cdot CR_{Bj}}{\sum_j \ell_{Bj}}. \quad \text{(A.6)}$$

and the new differential is

$$x_k^* = x_k \cdot \frac{\sum_j \ell_{kj}}{\sum_j e_{kj} \cdot CR_{kj}} \cdot \frac{\sum_j e_{Bj} \cdot CR_{Bj}}{\sum_j \ell_{Bj}}. \quad .\text{(A.7)}$$

But, using the existing values of x_i and y_j,

$$\sum_j e_{kj} \cdot CR_{kj} = \sum_j e_{kj} \cdot CR_B \cdot x_k \cdot y_j = CR_B \cdot x_k \cdot \sum_j e_{kj} \cdot y_j. \quad \text{(A.8)}$$

Substituting (A.8) into (A.7) we obtain

$$x_k^* = x_k \cdot \frac{\sum_j \ell_{kj}}{CR_B \cdot x_k \cdot \sum_j e_{kj} \cdot y_j} \cdot \frac{CR_{Bj} \cdot x_B \cdot \sum_j e_{Bj} \cdot y_j}{\sum_j \ell_{Bj}}. \quad \text{(A.9)}$$

Then substituting $x_B = 1$ and canceling the CR_B and x_k terms we obtain

$$x_k^* = \frac{\sum_j \ell_{kj}}{\sum_j e_{kj} \cdot y_j} \cdot \frac{\sum_j e_{Bj} \cdot y_j}{\sum_j \ell_{Bj}}, \quad \text{(A.10)}$$

which is the same as (A.3). Thus we see that, under the given assumptions, the Loss Cost Method and the Loss Ratio Method for determining classification differentials are algebraically equivalent.

Answers to Text Exercises

Chapter One

1.3 Proposal B

1.4 (a) Both choose X
(b) A chooses X and B chooses Y

1.5 (b) (i) Indifferent between A and B
(ii) Choose A

1.6 (a) 1/3
(b) 17/69 = .2464

1.7 15,109.54

1.8 4,875.05

1.9 390.46

1.10 No; the expected loss of utility value without insurance is 4272.17, which is less than the expected loss of utility with insurance of 4500.

1.11 (a) Utility with insurance = 13.16527
Utility without insurance = 13.16571
Do not buy.

(b) Utility with insurance = 13.16564
Utility without insurance = 13.16571
Do not buy.

1.13 (a) Yes (b) 1924.44 (c) 7,511,611.63

Chapter Two

2.5 512,000

2.6 80%

2.7 120,000 (the policy limit)

2.8 666.67

2.13 (a) 9,600
(b) 10,000
(c) 10,500

2.14 3.68

2.15 231.25

Chapter Three

3.2 Incurred

3.5

	Total	Case	IBNR
(a)	892.5	521.1	371.4
(b)	889.3	521.1	368.2
(c)	896.2	521.1	375.1

3.6 (a) 6,853
(b) 6,879
(c) 7,089

3.7 1,024,208

3.8 112.81

3.9 (a) 10,475
(b) 15,116
(c) 13,660

3.10 3,365

3.11 46,045

3.12 (a) 6,317
(b) 6,342
(c) 6,524

3.13

Method	Undiscounted	Discounted
Average	3,365	3,144
Mean	3,382	3,160

3.14 11,794,331

3.15 6,424

3.16 Yes, due to salvage and subrogation

3.17 (a) 14,018,299
(b) 13,256,992

3.18 (b) Yes: (i) Conservative case reserves
(ii) Salvage
(iii) Subrogation
(c) Larger
(d) 103.45
(e) 99.12

3.19 (a) 165
(b) Reserve = 110.87
(c) Discounted Reserve = 105.87

3.20 Total 1,429,849

CHAPTER FOUR

4.1 (a) 1,000
(b) 11,000
(c) 10,000

4.2 700

4.3 .63636

4.4 78 million

4.5 State A: 1/1/CY8
State B: 7/1/CY7

4.6 $2\frac{1}{12}$

4.9 290.22

4.10 The new gross rate is 650.74.

4.11 The gross premium is reduced to 474.05.

4.12 905.88

4.13 7.54%

4.16

Year	Earned Premium at Current Rates
CY4	3,286.35
CY5	3,498.44
CY6	3,689.27

4.17 1.276

4.18 Recommend that the differentials be recalculated, this time by the loss ratio method, reaching the following results:

Class	Differential
A	1.000
B	1.092
C	1.122
D	1.176
E	1.404
F	1.428
G	1.560

Reason: Loss Ratio method less affected by cross-variable heterogeneity (see page 134)

4.19 113.31

4.20 198.94

4.21 207.29

4.22 20.4%

4.23 132.76

4.24 253.72

4.25 +14.6%

4.26

Terr	Adopted Rel
A	1.784
B	1.000
C	1.876

4.27

Class	New Rate
1	$111.55
2	127.72
3	153.94

Chapter Five

5.1 Schedule rating is used initially to reflect the expected reduction in losses. Once enough history is reflected in the insured's experience, the reduction would be captured in the experience rating and be removed from the schedule rating credits to avoid double-counting.

5.2 428,452

5.3 261,892

5.4 ILF for 1,000,000 limit: 2.115
ILF for 2,000,000 limit: 2.387

5.5 (a) ILF = 1.97
(b) Adjusted ILF = 2.08

5.6 $500 deductible: 0.957
$1,000 deductible: 0.921

5.7 $0 deductible: 1.284
$1,000 deductible: 0.785

5.8 0.494

5.9 11.6%

5.10 Primary insurer : 1.3%
Reinsurer : 41.4%

5.11 Losses above $500,000 and $2,000,000 can have different loss development patterns and trends.

5.12 Excess of loss, stop-loss, and catastrophe

5.13 1,500,000

5.14 (a) 27,000
(b) 300,000

5.15 132,500,000

Index

About The Text

This text provides a basic foundation of knowledge concerning two fundamental building blocks of property/casualty actuarial work: ratemaking and loss reserving. Although the material is of property/casualty origins, the methods presented have potential application in other insurance areas including health insurance and risk management. The text contains a number of worked examples and end-of-chapter exercises.

The fourth edition reverses the order of chapters three and four from previous editions. The estimation of the ultimate claim payments is a necessary first step in both the loss reserving process and ratemaking process. Determining the ultimate losses is more comprehensively covered in the loss reserving chapter, and the ratemaking process often relies on the estimates of ultimate losses determined in the loss reserving process. As a result, the loss reserving chapter now comes before the ratemaking chapter.

The frequency and severity section of the loss reserving chapter has been revised to demonstrate the closure method of estimating ultimate losses.

The chapter on intermediate topics has been updated to include deductible pricing, as this alternative approach to the ratemaking in chapter four is typically used for pricing various deductible options.

Finally, the fourth edition has been updated to reflect industry changes and includes even more exercises than previous editions.

About The Authors

Robert L. Brown, Ph.D., FSA, FCIA, ACAS, graduated from the University of Waterloo in 1971 with a BMath degree. He added a MA in Gerontology in 1994 (Waterloo) and a Ph.D. (Gerontology) from Simon Fraser University in 1997. Rob is a Fellow of the Canadian Institute of Actuaries, a Fellow of the Society of Actuaries and an Associate of the Casualty Actuarial Society.

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